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DYNAMIC ANALYSIS OF A LAUNCH AND RECOVERY SYSTEM FOR A DEEP SUBMERSIBLE

by

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Submitted to the Department of Naval Architecture and Marine Engineering in partial fulfillment of the requirements for the degree of Master of Science in Ocean Engineering.

ABSTRACT

An analysis of the motion of a docking platform for a deep submersible is presented. The problem is defined as a docking platform, or cradle, suspended beneath a surface vessel by elastic elements, composed of cable, chain, or similar material. The analysis attempts to predict the motion of the cradle in response to sinusoidal motion of the surface vessel.

The physical system is mathematically modeled as a non-linear differential equation of the form:

$$M \frac{d^2y}{dt^2} + b\left|\frac{dy}{dt}\right|\frac{dy}{dt} + Ky = KXo \sin(\omega t)$$

This assumes a single degree of freedom system, which models the cradle as a mass connected by a linear spring to a vessel moving vertically in a sinusoidal fashion. The damping is non-linear and is proportional to velocity squared.

This problem is solved for transient as well as steady-state conditions by numerical techniques. An equivalent linear differential equation is proposed. The results are applied to four examples. Three of them are directly related to a possible application of the launch and recovery system to the new deep submersible Sea Cliff and her support ship Lulu. The fourth example gives an account of the application of the results to the salvage operation of the deep submersible Alvin.

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I. PROBLEM STATEMENT

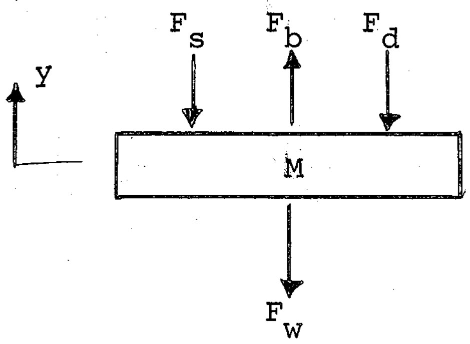
Launch and recovery operations for a deep submersible are extremely sensitive to surface conditions. Since it is desirable to operate a submersible during relatively poor weather conditions, recent design concepts of launch and recovery systems have stressed penetrating the air-sea interface as quickly as possible. One such concept suggests lowering a cradle by chain or cable to a distance below most of the surface disturbances. At a depth of, perhaps, 100 feet it would be possible for a submersible to dock or depart from a cradle without the threat of poor surface conditions. The major concern in such an operation would be the motion of the cradle itself. Since the cradle must be connected to the surface vessel, it must also respond to the motions of the support ship. In particular, short period, high acceleration motions would prohibit a submersible from conducting safe operations. The analysis of possible cradle motions is the primary concern of this investigation.

II. FORMULATION OF THE DIFFERENTIAL EQUATION

The motions of a cradle beneath a ship are extremely complex. The ship can heave, roll, pitch, and move horizontally. From the point of view of a submersible involved in a docking operation, the most dangerous cradle motions are vertical ones of high frequency. Vertical cradle motions can be excited by heave of the surface ship, or by pitch and roll if the cradle is not attached at the center of rotation of the surface vessel.

To simplify this problem, it is assumed that the dominant motion of the cradle would be vertical. It is also assumed that any horizontal velocities of the surface ship would be so low that the cradle would hang nearly vertically beneath the surface vessel. The final assumption is to consider the cradle motion as the response of a single degree of freedom system, driven at its point of attachment to the surface vessel. In other words, it is assumed that the motion of the cradle would not affect the motion of the ship. This assumption is substantiated later by a specific example. With these assumptions, the problem is reduced to one of a practical size.

To proceed with the solution, a differential equation and hence a free body diagram are necessary. In the one that follows, the y-axis is positive upwards, and the instantaneous displacement and velocity of the cradle are both positive.



F_w = Weight of cradle

F_b = Buoyancy of cradle

F_d = Drag force

F_s = Spring force

M = Virtual mass of cradle

Free Body Digram

FIGURE 1

The summation of external forces can be expressed as follows:

$$F_{ex} = F_b - F_w - F_s - F_d = M \frac{d^2y}{dt^2}$$

Before writing the differential equation, each contributing member must be examined.

<u>INERTIAL</u> <u>TERM</u>: The mass of the cradle has two complicating factors. First, during actual conditions it will have two vastly different states depending on whether the submersible is on board or not. Second, the mass of the cradle must include the mass of entrained water that is accelerated by cradle motion. It is assumed that the entrained water will be reasonably constant (Ref. 6,9), therefore we can conclude that the inertial term has a mass associated with it of the form:

$$M = m_{cradle} + m_{added} + m_{sub} \text{(when appropriate)}.$$

Of course, the added mass term will be different for the two cradle states.

SPRING FORCE: Any material which is used to suspend the cradle, whether it is steel or synthetic cable, or steel chain, can act as a spring. The more rigid the material the higher the spring constant. This analysis does not specify any particular type of material, but it does assume that the spring is linear, that is the stretch is proportional to the load.

In the case of cable or chain, the spring constant is given by:

$$K = EA/L \text{ in lbs/ft},$$

where: E = Modulus of elasticity,
A = Cross sectional area of cable or chain,
L = Length of spring element.

Most likely, the cradle will be suspended by three or more cables. Springs connected in such a parallel arrangement can be represented by an equivalent single spring with a constant equal to the sum of the spring constants of all elements.

Note that the spring constant K is inversely proportional to the length of line. Thus if the primary spring in a system were a cable, then the dynamic properties would be a function of the depth the cradle is below the surface. This dependence on depth could be circumvented by the use of a short spring element connected in series to a relatively stiff chain or cable. For two springs connected in series an equivalent spring can be used in the mathematical model which is given by the equation:

$$1/K_{eq} = 1/K_1 + 1/K_2.$$

Thus springs in series combine like parallel electrical resistances and conversely. The spring element could be connected between the cradle and chain, and therefore the spring constant would be nearly independent of depth.

For the purpose of this analysis the spring constant is assumed to be constant. This work attempts to describe the motion at any particular spring constant, but it does not attempt to describe the changing dynamic conditions as the cradle is hauled upward.

It is through the spring element, whatever its construction, that the excitation is transmitted from the surface. In this analysis it is assumed that the surface vessel is heaving vertically in a sinusoidal fashion. This situation can be visualized as in the following diagram:

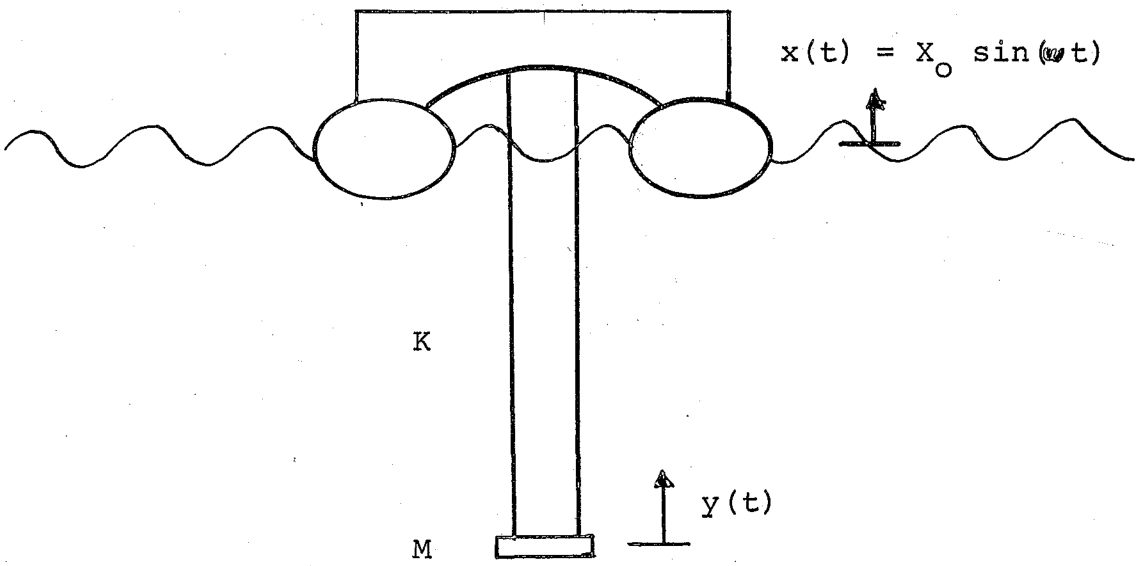


FIGURE 2, Physical System

The tension in the spring at any instant is therefore given by:

$$T = K(x(t) - y(t)),$$

where x and y are taken to be zero when the system is at rest. The spring force can now be represented as:

$$F_{spring} = K(X_o \sin(\omega t) - y(t)).$$

DRAG FORCE: Drag forces are predominantly due to fluid drag on the cradle or on the cradle and submersible together. For the purpose of this analysis the drag on the chain or cable is assumed to be negligible in comparison.

In determining drag coefficients for this problem it is necessary to consider drag due to viscous effects and drag due to pressure differences, also known as form drag. References generally agree that for hard-edged geometric shapes, drag coefficients are independent of Reynolds number. For example, the drag on a square, flat plate normal to the free stream is given by:

$$\text{Drag} = \tfrac{1}{2}\rho S C_D V^2,$$

where: C_D = 1.1 and is independent of Reynolds number and therefore velocity (Refs. 2,5,8),
S = Cross sectional area,
ρ = Density of fluid,
V = Velocity of fluid.

In such cases as this the drag is due primarily to form drag and very little to viscous effects. The form drag is attributable to pressure differences on opposite surfaces of the plate. The cradle would conceivably be a sharp-edged geometric shape, and hence the drag coefficient would be assumed constant. However, a submersible (such as Sea Cliff) is not hard-edged, but is a rather bulky ellipsoid with many appendages. Frictional drag is a significant segment of the total drag. Viscous drag coefficients are dependent on Reynolds number. If the submersible is thought of as a sphere, then it is known that the drag coefficient for a sphere at Reynolds numbers greater that 10^6 change rather slowly. From the point of view of the differential equation the drag term is particularly important when it is large compared to the other terms of the equation. This occurs at relatively high velocities and consequently at relatively high Reynolds numbers. To assume a constant drag coefficient gives a reasonable approximation of the physical system. For example, if the submersible is compared to a 12-foot diameter sphere, then at 1 ft/sec the Reynolds number in sea water is 10^6. From model studies conducted by Woods Hole Oceanographic Institution, the drag coefficient for Alvin in vertical motion is 1.0. (Ref. 10)

In this analysis the drag coefficient is assumed to be a constant value which must be determined for each application. Note, however, that the drag is proportional to velocity squared. This squared term is a non-linear element in the final differential equation.

STATIC FORCES: As a last step before writing the differential equation we must consider the static forces of buoyancy and weight. The cradle must have a net negative buoyancy, or else it will float to the surface. If the differential equation is written using the origin of coordinates as the static equilibrium position of the cradle then the static forces do not enter into the differential equation. It must still be kept in mind that if the dynamic situation becomes such that the cable becomes totally relaxed, then it cannot resist further relaxing motions. This situation is significant from a practical point of view, because it means that the cable has gone slack. This event is considered in more detail later.

With the foregoing analysis in mind we can now write the differential equation:

$$M \frac{d^2y}{dt^2} + b\left|\frac{dy}{dt}\right| \frac{dy}{dt} + Ky = KX_o \sin(\omega t)$$

where: $b = \frac{1}{2}\rho S C_D$

A literature search revealed no closed form solution to this non-linear equation. However, a U.S. Navy report (Ref. 4) considers the problem of lowering large weights on cables to depths up to 20,000 feet. This report uses a differential equation that takes into account the length of time for a force applied at the surface to propagate down the cable. For 8-inch circumference polypropylene line, loaded to 10% of its breaking strength, this is about 3000 ft/sec. For very short lengths, as in the problem considered in this report, the propagation time is very small in comparison to the period of motion, and need not be considered. The damping in the Navy report was also estimated to be proportional to the square of the velocity. To avoid the problem of this non-linearity, they estimated an equivalent linear damping which dissipated the same average power, but admitted that it could be in error by as much as 20% in predicting amplitudes.

III. REDUCTION OF THE DIFFERENTIAL EQUATION TO NON-DIMENSIONAL FORM

A generalized solution is most easily presented in non-dimensional form. To aid in non-dimensionalizing, it was helpful to compare this non-linear equation to its closest linear relative. The following differential equation is linear, has a known closed form solution, and in every respect, except the drag term, resembles the non-linear problem:

$$M \frac{d^2y}{dt^2} = -C \frac{dy}{dt} - Ky + KX_o \sin(\omega t).$$

The response of this linear system to external excitation is completely known, and is known to depend strongly on the nominal natural frequency which is defined as:

$$\omega_o = \sqrt{K/M}.$$

Peak amplitude response occurs at or slightly less than this frequency, depending on the amount of damping. The solution to this equation is presented in greater detail in a later section, but it is sufficient for now to use this concept of natural frequency.

In the non-linear equation the response will also be strongly dependent on the same definition of natural frequency, but will differ in some respects due to the non-linear damping. Using this concept of natural frequency we can proceed to non-dimensionalize both the linear and the non-linear differential equations.

Let the non-dimensional response be defined as an amplitude ratio:

$$Y = y/X_o.$$

The non-dimensional response is the ratio of the cradle amplitude to the maximum amplitude of the excitation.

Let the non-dimensional time be defined as the ratio of real time to the natural period of oscillation.

$$\text{One period} = 2\pi/\omega_o,$$

$$T = t/(2\pi/\omega_o).$$

The expressions $y = YX_o$ and $t = T(2\pi/\omega_o)$ can now be substituted into the differential equations. The non-linear equation:

$$M\frac{d^2y}{dt^2} = -b\left|\frac{dy}{dt}\right|\frac{dy}{dt} - K(y - X_o \sin(\omega t)),$$

now becomes:

$$\frac{MX_o\omega_o^2}{4\pi^2}\frac{d^2Y}{dT^2} = \frac{-b\omega_o^2 X_o^2}{4\pi^2}\frac{dY}{dT}\left|\frac{dY}{dT}\right| - KX_o(Y - \sin(2\pi\frac{\omega}{\omega_o}T)),$$

By defining the damping coefficient for the non-linear equation as $B = bX_o/M$ and recognizing that $K/M = \omega_o^2$, the final form of the non-linear differential equation is reached:

$$\frac{d^2Y}{dT^2} = -B\left|\frac{dY}{dT}\right|\frac{dY}{dT} - 4\pi^2(Y - \sin(2\pi\frac{\omega}{\omega_o}T)).$$

The same non-dimensionalizing substitutions can be made in the linear equation with the following results.

$$\frac{d^2Y}{dT^2} = \frac{-4\pi C}{2\omega_o M}\frac{dY}{dT} - 4\pi^2\left(Y - \sin\left(\frac{2\pi\omega T}{\omega_0}\right)\right).$$

The quantity $C/2\omega_o M$ is traditionally defined as the damping ratio:

$$\xi = C/2\omega_o M.$$

The non-dimensional form of the linear differential equation is:

$$\frac{d^2Y}{dT^2} = -4\pi\xi\frac{dY}{dT} - 4\pi^2\left(Y - \sin\left(\frac{2\pi\omega T}{\omega_0}\right)\right).$$

It will be of interest later to have a non-dimensional form of the homogeneous equations for both the linear and non-linear cases. As there is no forcing function it is necessary to redefine the dimensionless displacement. Instead of $Y = y/X_o$, let $Y = y/y_o$ be the ratio of the displacement to the initial displacement which must be prescribed as an initial condition. The result of an initial displacement is a periodic motion at the natural frequency with decaying amplitudes. In the linear case, the decay is exponential.

The dimensionless homogeneous equations are:

(a) Non-Linear

$$\frac{d^2Y}{dT^2} + B\left|\frac{dY}{dT}\right|\frac{dY}{dT} + 4\pi^2 Y = 0,$$

(b) Linear

$$\frac{d^2Y}{dT^2} + 4\pi\xi\frac{dY}{dT} + 4\pi^2 Y = 0,$$

Non-dimensional equations are used exclusively in the following analysis. All presentation of results is given in non-dimensional terms.

IV. METHOD OF SOLUTION

As mentioned before, the linear equation has a known closed form solution for both steady-state as well as transient conditions. The non-linear equation does not have a known closed form solution for the driven case, but it does have an accurate linearized solution for the homogeneous equation with an initial displacement.

The solution of the driven non-linear equation had to be found by numerical techniques. The 1130 Computer Facility in M.I.T.'s Mechanical Engineering Department has a fourth order Runge-Kutta program which can numerically integrate a system of first order differential equations. This particular second order equation can be expressed as two first order equations as shown below, and these can be solved by the program:

$$\frac{d^2Y}{dT^2} = -B\left|\frac{dY}{dT}\right|\frac{dY}{dT} - 4\pi^2\left(Y - \sin\left(2\pi\frac{\omega}{\omega_o}T\right)\right).$$

Let $Y = Y1$, and $\frac{dY1}{dT} = Y2$.

Then $$\frac{dY2}{dT} = -B|Y2|Y2 - 4\pi^2\left(Y1 - \sin\left(2\pi\frac{\omega}{\omega_o}T\right)\right).$$

The single second order equation in one dependent variable Y has been expressed as two first order equations in Y1 and Y2, which can be solved by numerical integration.

PROGRAM DESCRIPTION: The Runge-Kutta program was converted for use on the SDS Sigma-7 at Woods Hole Oceanographic Institute. The program was used in two forms. The first was the original M.I.T. program. In this form the computer integrated the differential equations in small increments of time from zero out to a prescribed limit. The output was in printed form, and it included Y1, Y2, dY1/dT, dY2/ dT, and T.

In the second form, the M.I.T. program was converted to plot Y1 only. Figure 9 is an example of this plotted output. Listings of both programs and a complete description of their use is given in Appendix II.

ACCURACY OF NUMERICAL METHOD: Error control in numerical procedures is of prime importance. Some numerical schemes for solving propagation problems have inherent instabilities. This particular program was extremely well-behaved in solving the linear and non-linear equations considered in this paper.

The program has written into it an error test. If the time increment appears too large to insure that the error falls within limits prescribed by the user, then the program automatically halves the increment.

To demonstrate the accuracy of this numerical technique, the exact solution given on page 31 for the linear nonhomogeneous differential equation with a non-zero initial displacement is shown in Figure 3. The same differential equation and initial conditions were integrated out and plotted, as shown in Figure 4. For all practical purposes they are identical. A transparency of the exact solution is provided as an overlay. Since the non-linear equation is essentially the same from a computational point of view, then a great deal of confidence can be placed in the accuracy of the numerical procedure.

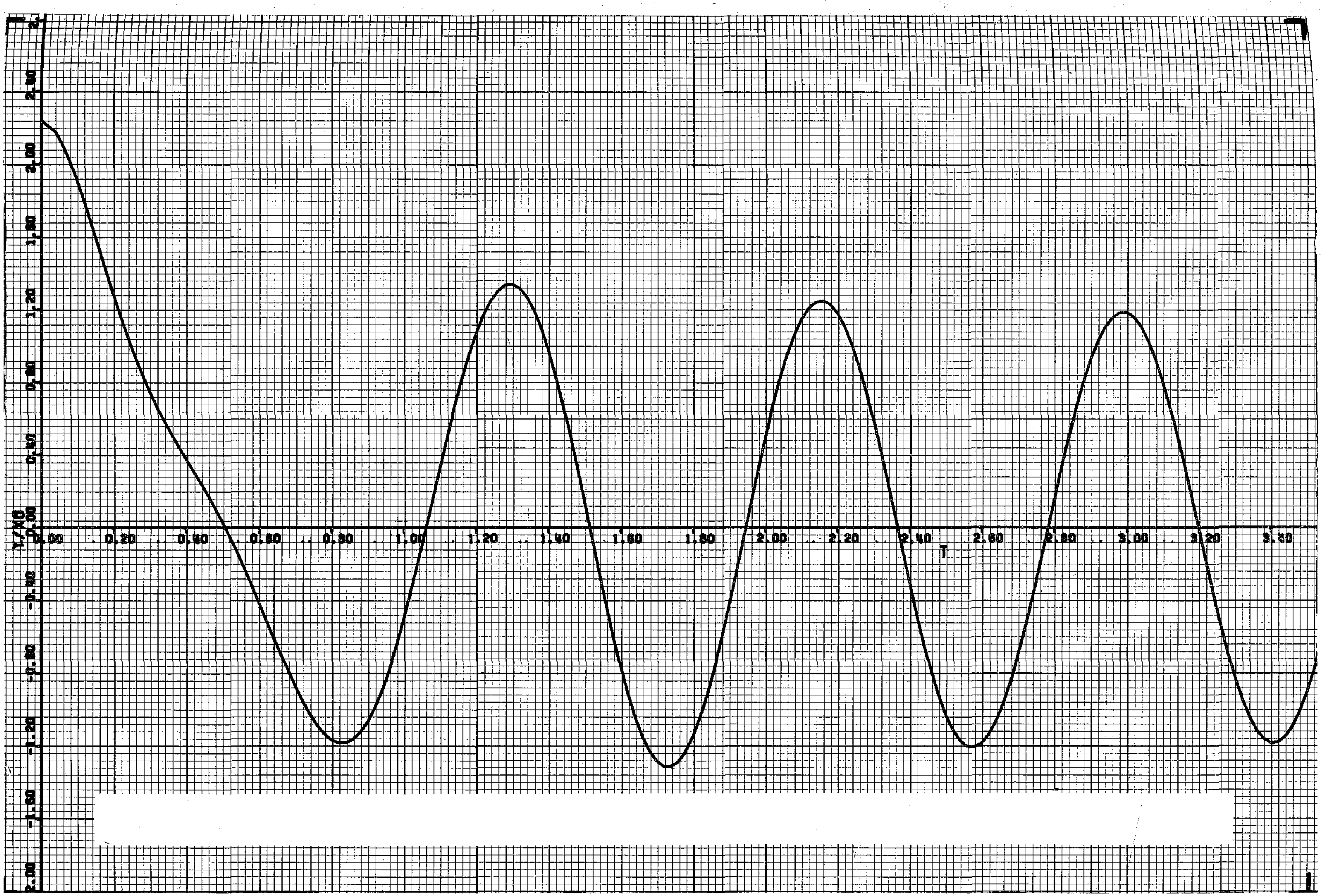


Figure 4. Runge-Kutta Integration of Linear Differential Equation

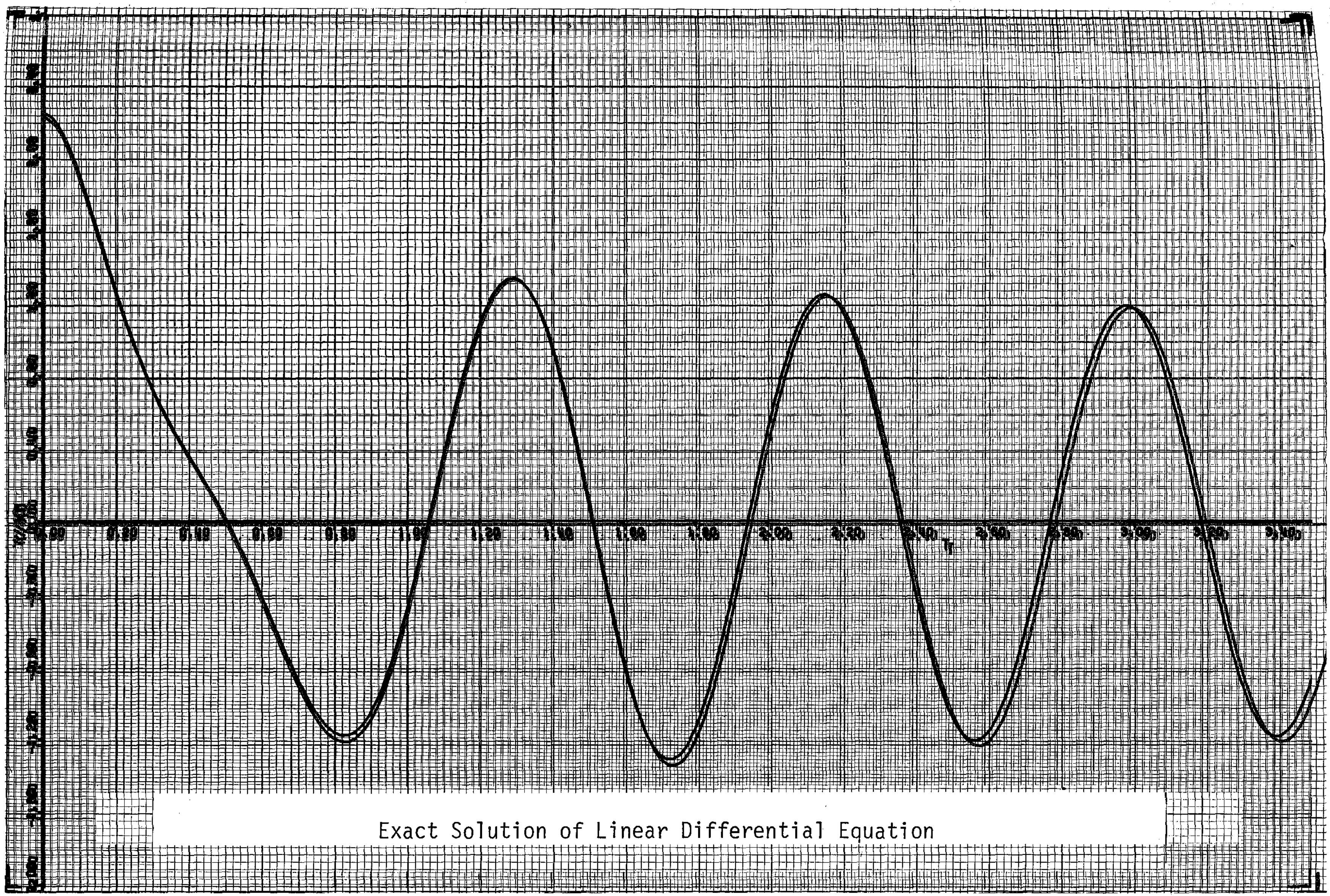


Figure 4. Runge-Kutta Integration of Linear Differential Equation

V. GENERAL RESULTS

A mechanical system which is driven by a sinusoidal forcing function will respond in a periodic manner at the same frequency of the driving force. The amplitude of response is controlled primarily by the damping in the system, and by the existence of any natural frequencies. In general, larger amplitudes result from low damping and nearness to a natural frequency.

Two mechanical systems have been considered in this investigation. They are identical in all respects, except that one has damping proportional to velocity squared and the other to velocity. It is convenient to examine each of these equations in terms of steady-state response and transient response. All solutions are presented in non-dimensional form.

STEADY-STATE RESPONSE: In both mechanical systems the steady-state response is a periodic motion which has constant maximum amplitudes. All transient effects have died out; that is, initial conditions do not affect steady-state response. The steady-state response for the linear equation is known, and can be computed using the following equations (Ref. 1):

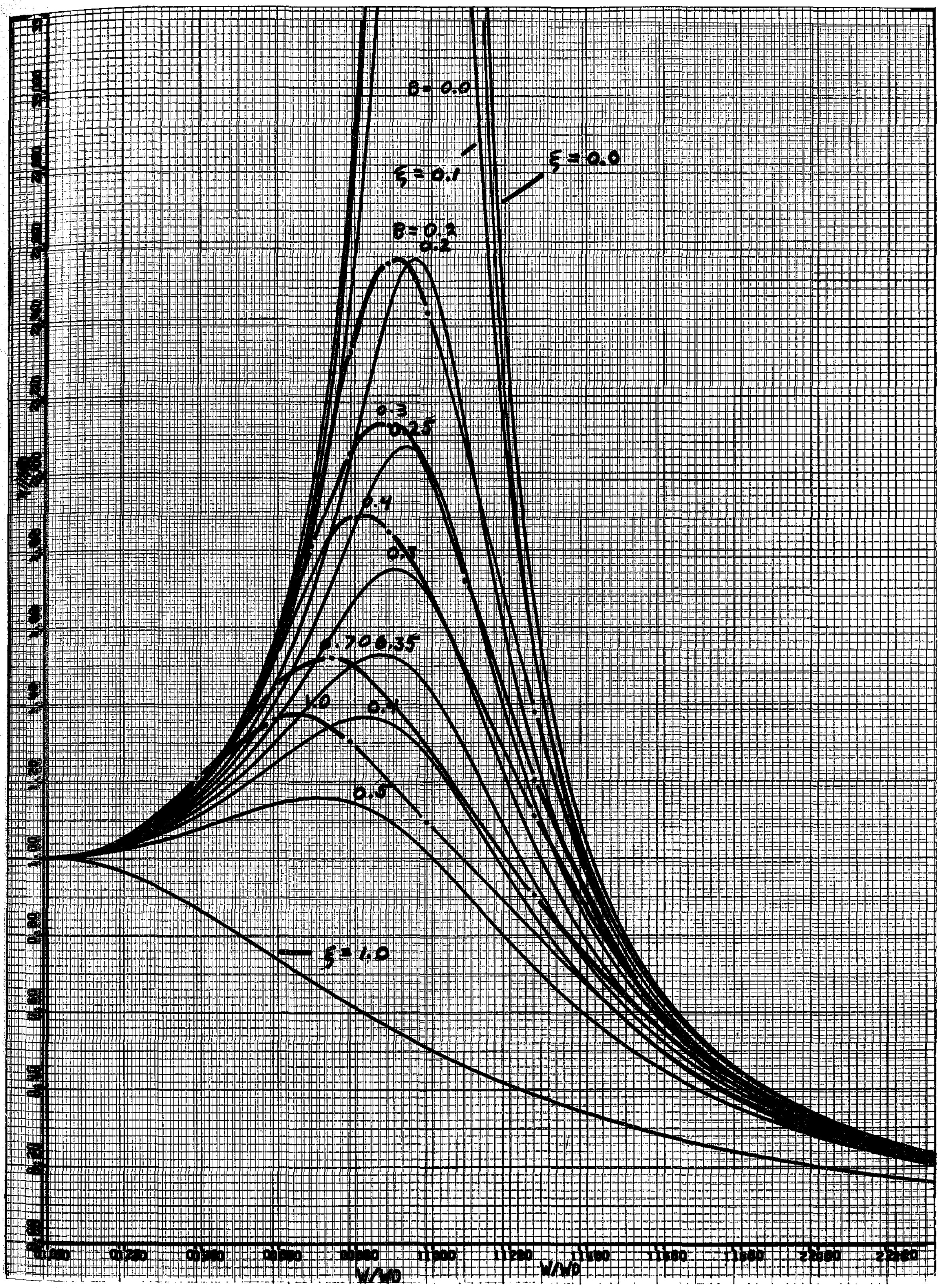
$$(1) \quad Y_{max} = \frac{1}{\left(\left(1 - \frac{\omega^2}{\omega_o^2}\right)^2 + 4\xi^2 \frac{\omega^2}{\omega_o^2}\right)^{1/2}}$$

$$Y_{max} = y_{max}/X_o$$

$$(2) \quad \tan(\psi) = \frac{2\xi \frac{\omega}{\omega_o}}{1 - \frac{\omega^2}{\omega_o^2}}$$

Figure 6 is a graphical presentation of Equation 1 for several different damping ratios. The ordinate y_{max}/X_o is called transmissibility and is the ratio of peak cradle amplitude to peak amplitude of the driver or ship. The abscissa is the frequency ratio ω/ω_o, where ω is the driving frequency in radians per second and $\omega_o = \sqrt{K/M}$. For example, if $\omega/\omega_o = 1.0$ and $\xi = 0.25$, then $Y_{max} = y_{max}/X_o = 2.0$. The cradle motion is twice that of the ship.

Figure 5 is a transmissibility plot for the non-linear differential equation. Since no closed form solution is known this curve had to be constructed using the numerical results of the computer program. For each point on this plot it was necessary to integrate the differential equation using the Runge-Kutta program. For each computer run it was necessary to specify ω/ω_o, the damping coefficient, and the initial conditions. By intelligently estimating the



Transmissibility vs. Frequency Ratio - Non-Linear Equation

Figure 6. Transmissibility vs. Frequency Ratio - Linear Equation

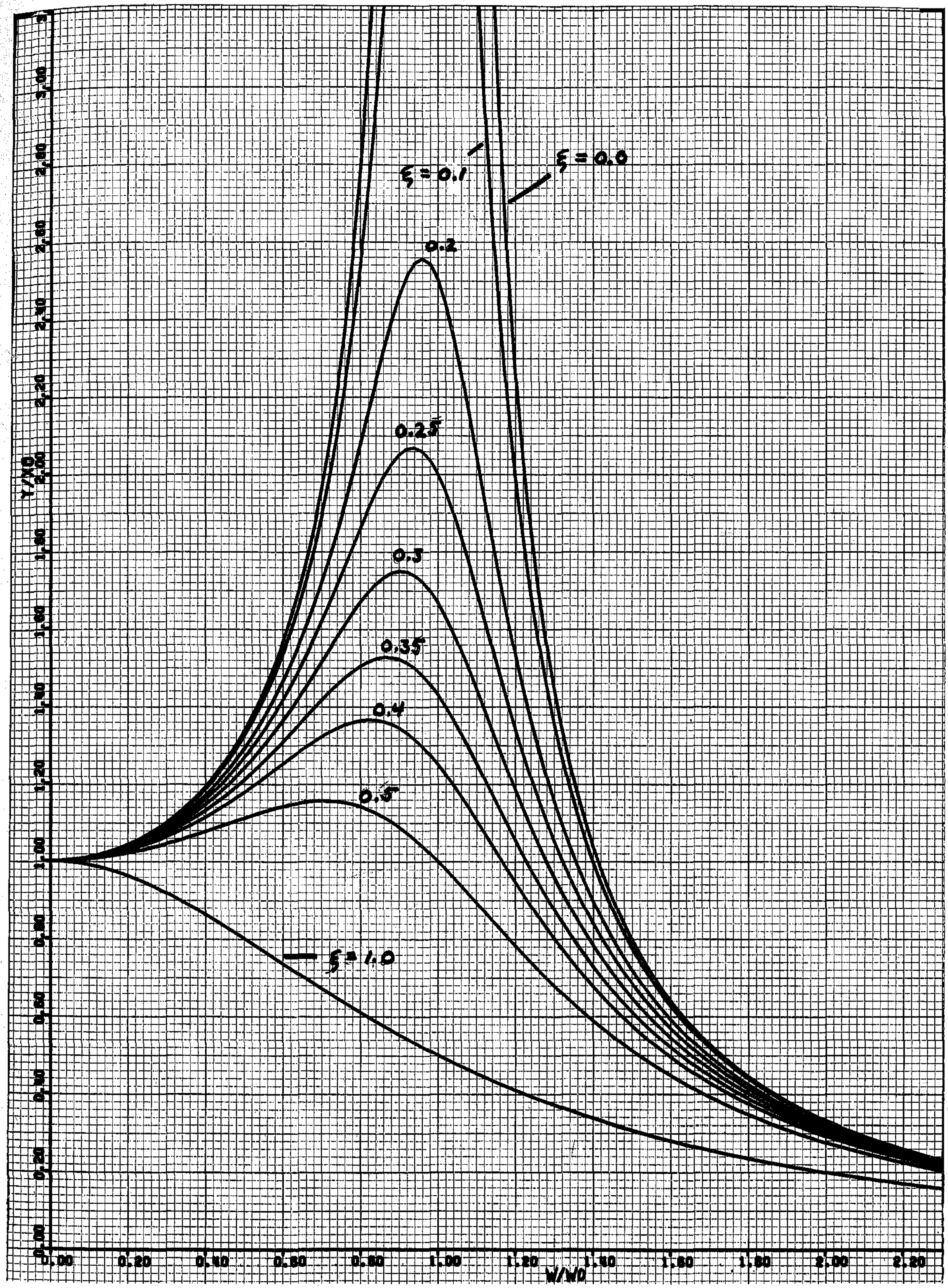


Figure 6. Transmissibility vs. Frequency Ratio - Linear Equation

initial conditions, it was possible to put the differential equation into motion very near to steady-state. After any remaining transients died out, the peak amplitude of the steady-state motion could be obtained. All runs were initiated at T = 0. Steady-state was always achieved, for all practical purposes, by T = 5.0.

In addition to transmissibility, it is necessary to know the phase difference between the ship and the cradle. Again this is known for the linear equation, but it is not known for the non-linear. Equation 2 gives the governing equation for the linear case. Figure 8 is a plot of this phase relationship. Note, the motion of the cradle always lags the motion of the ship. At frequency ratios much less than 1.0 the lag is very small. At resonance the lag is 90° and for frequency ratios much larger than one, the phase lag asymtotically approaches 180°. As with transmissibility, phase is a function of frequency ratio and damping. The phase plots are given for the same damping ratios as the transmissibility curves.

For the non-linear equation, it was possible to deduce the phase lag from the results of the integration of the differential equation. The computer integration begins at T = 0.0. The period of one cycle is ω_o/ω in dimensionless time. At some time later, after which steady-state has been achieved, the time lag between driving function and cradle response can be calculated. For instance, some fraction of a full period ellapses between the time that the driving term is zero and the time that the response is zero. This fraction multiplied by 360° in one period gives the phase lag.

For convenience, transparencies of the non-linear phase and transmissibility have been presented as overlays to show the influence of the non-linear damping.

<u>TRANSIENT</u> <u>RESPONSE</u>: The following equations give the general response of the linear system (Ref. 1):

$$y = e^{-\xi \omega_o t}\left[A1 \sin\left(\omega_o\sqrt{1-\xi^2}\,t\right) + A2 \cos\left(\omega_o\sqrt{1-\xi^2}\,t\right)\right]$$

$$+ \frac{X_o \sin(\omega t - \psi)}{\left(\left(1 - \omega^2/\omega_o^2\right)^2 + 4\xi^2\left(\omega^2/\omega_o^2\right)\right)^{1/2}}.$$

In dimensionless form this is:

$$Y = e^{-\xi 2\pi T}\left[\frac{A1}{X_o} \sin\left(2\pi\sqrt{1-\xi^2}\,T\right) + \frac{A2}{X_o} \cos\left(2\pi\sqrt{1-\xi^2}\,T\right)\right]$$

$$+ \frac{\sin\left(2\pi\frac{\omega}{\omega_o}T - \psi\right)}{\left(\left(1 - \frac{\omega^2}{\omega_o^2}\right)^2 + 4\xi^2\left(\frac{\omega^2}{\omega_o^2}\right)\right)^{1/2}}.$$

The first term in this solution is the transient, and the second term is the steady-state.

The constants A1 and A2 are selected to give the desired initial conditions for displacement and velocity.

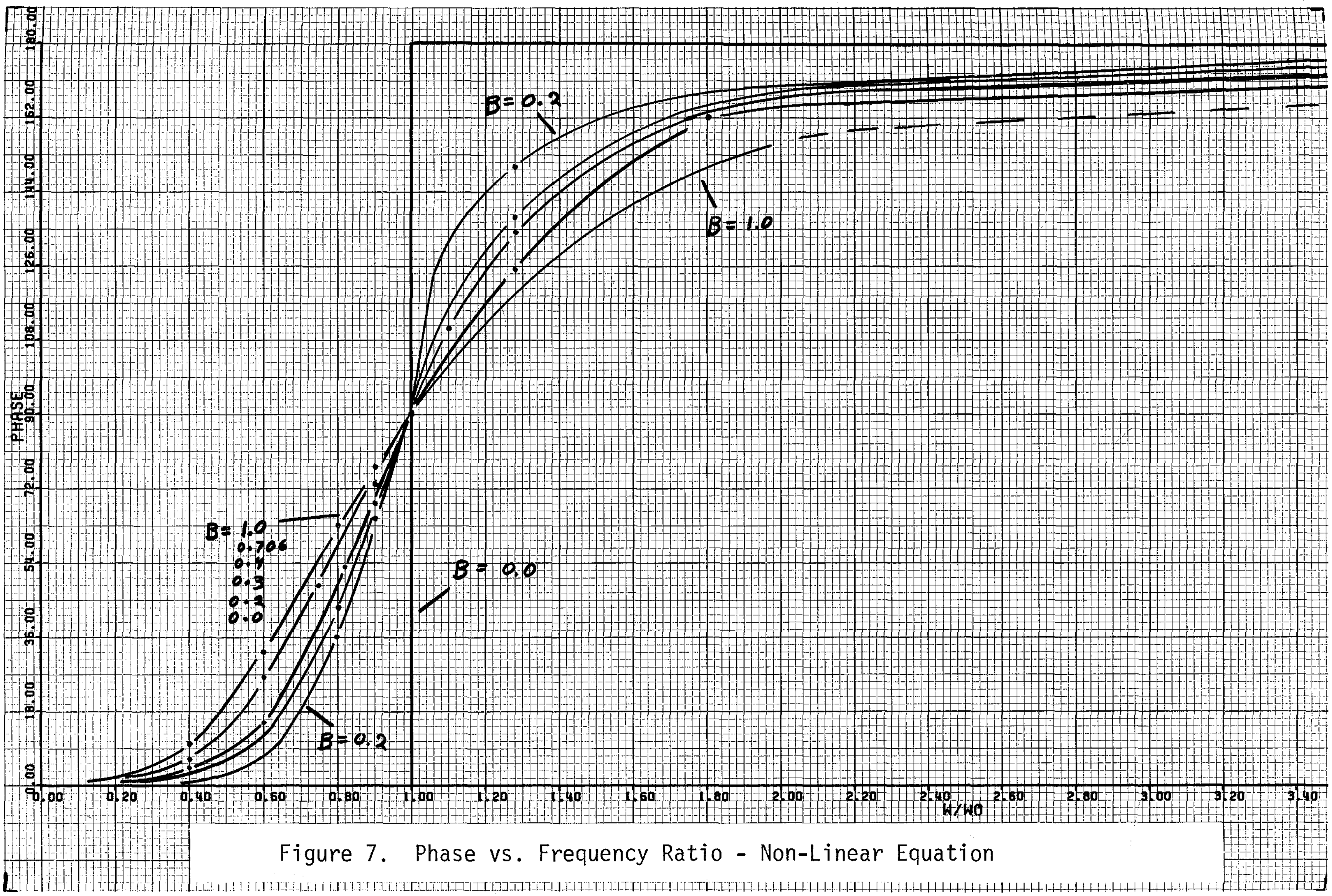
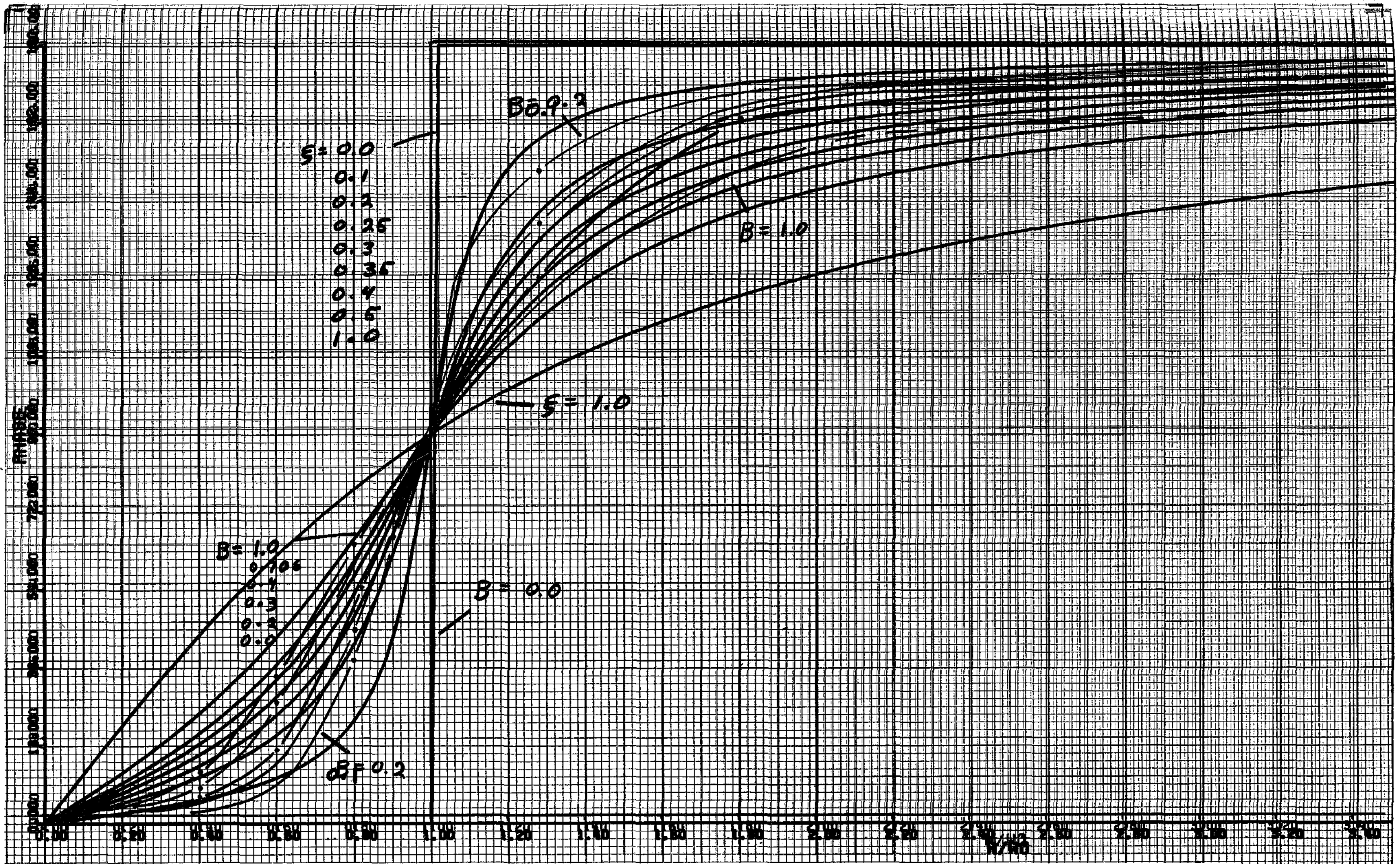


Figure 7. Phase vs. Frequency Ratio - Non-Linear Equation



Phase vs. Frequency Ratio - Non-Linear Equation

Figure 8. Phase vs. Frequency Ratio - Linear Equation

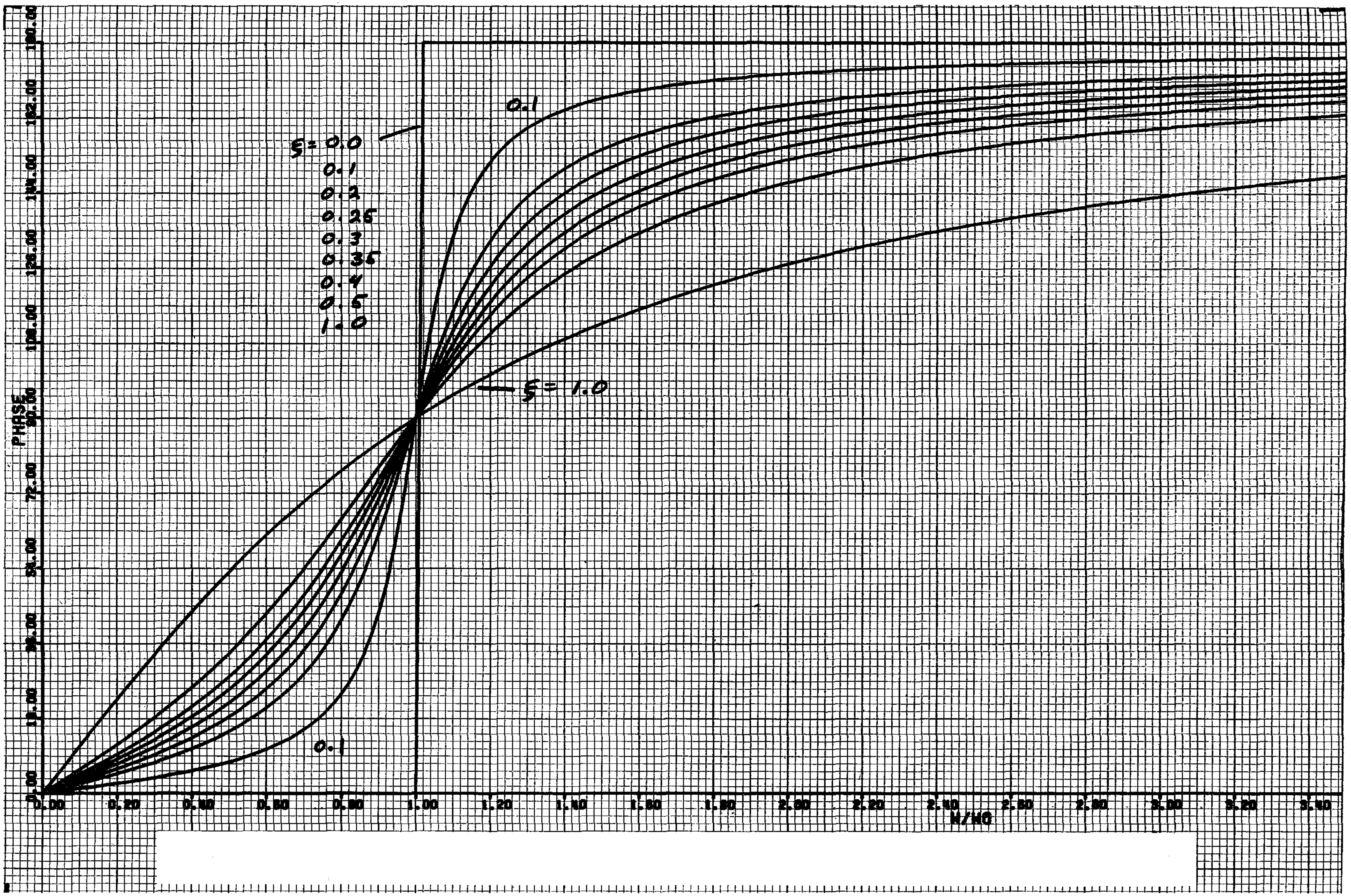


Figure 8. Phase vs. Frequency Ratio - Linear Equation

For Figure 3, $A1/X_o$ and $A2/X_o$ were specified as 1.767 and 3.2533 to give an initial displacement of 2.24 and a zero initial velocity. The other important parameters in the example are $\omega/\omega_o = 1.2$ and $\xi = 0.3$. As can be seen in the figure, the transient decay occurred in the first few cycles, and thereafter the solution was steady-state only.

Some information on the transient phenomena of the non-linear system has been presented by Minorsky in his book, "Non-Linear Mechanics" (Ref. 7). The solution he gives predicts the envelop of decay for the homogeneous non-linear differential equation with an initial displacement. The following equation is the decay envelop in dimensional terms:

$$y(t) = \frac{y_o}{1 + \frac{4b\omega_o y_o}{3\pi M} t},$$

where: $\omega_o = \sqrt{K/M}$,
$b = \frac{1}{2}\rho S C_D$,
y_o = Initial cradle displacement.

This can be put in non-dimensional form as follows:

Let $Y = y/y_o$, and $T = t/(2\pi/\omega_o)$.

This yields: $$Y = \frac{1}{1 + \frac{4b2\pi y_o}{3\pi M} T} = \frac{1}{1 + \frac{8by_o}{3M} T}.$$

Y is now the solution to the homogeneous non-linear equation. This solution was obtained by a linearizing procedure and is not exact. However, the accuracy is excellect, as shown in

Figure 9. The predicted envelop and the solution found by the numerical procedure agree very closely. In this case, $by_o/M = 0.3$.

It might be thought that such a linearized solution for the transient case could be extended to the steady-state. In particular, a common technique with non-linear damping is to pick the equivalent linear damping which dissipates the same power. In one reference (Ref. 4) this solution was used and was admitted to be as much as 20 % in error. The numerical technique used in this study is far superior to any such approximation and consequently none was used though several were investigated.

<u>EQUIVALENT</u> <u>LINEAR</u> <u>SYSTEM</u>: With results of this study it is possible to find the best linear equivalent for any specific damping. By using the non-linear transmissibility curve in the form of a transparent overlay as in Figure 6, it is possible to pick the linear equivalent damping, which gives the same transmissibility at the same frequency ratio. For example, a non-linear damping coefficient of $B = 0.294$ and a linear one of $\xi = 0.25$, both yield a transmissibility of 2.0 at $\omega/\omega_o = 1.0$.

The steady-state solution for this non-linear case, and

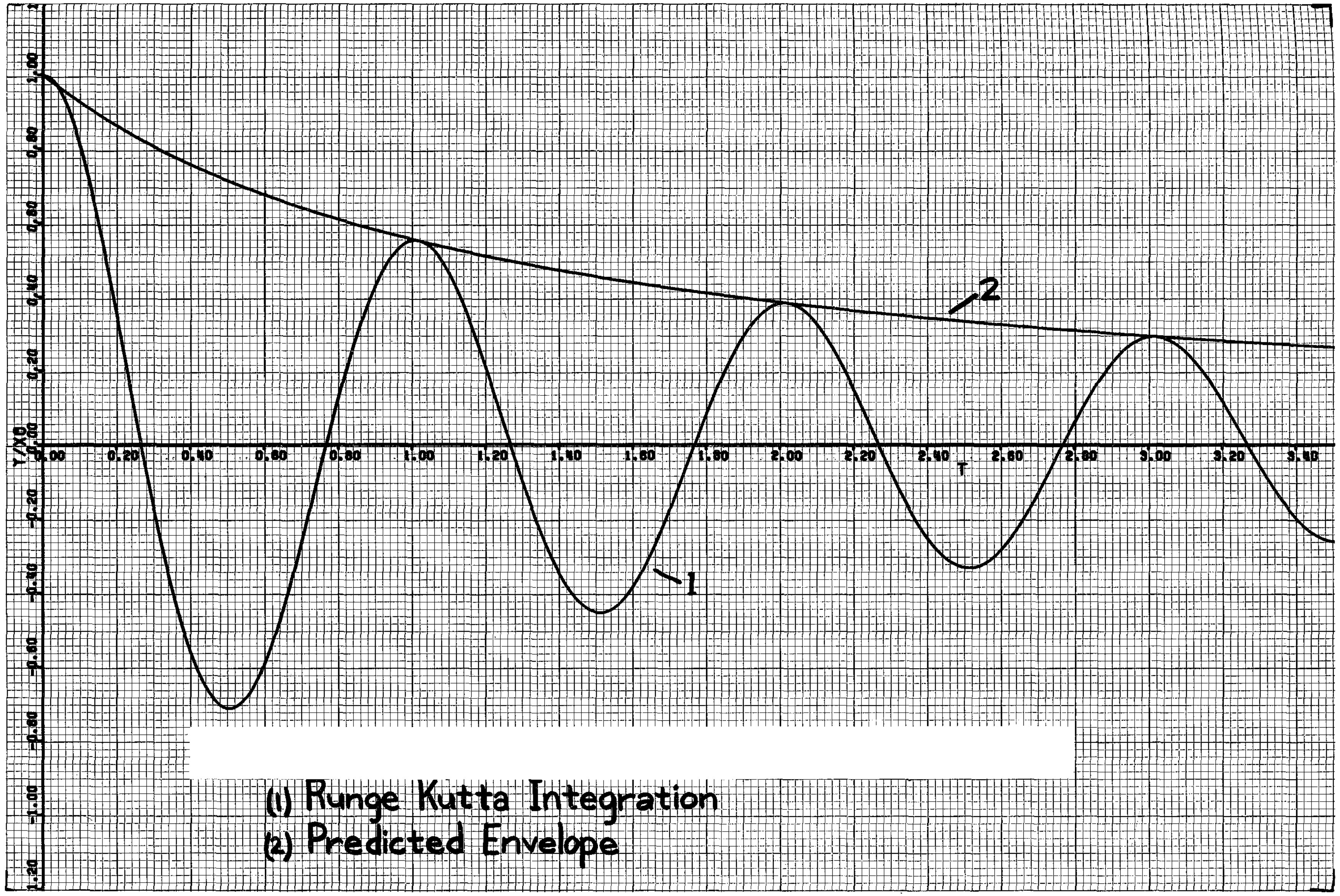


Figure 9. Transient Decay of Non-Linear System

its linear equivalent are plotted in Figure 10. In this particular example, they are almost identical. Note however that the solution to the linear problem is a true sinusoid, while the non-linear solution is not. In addition for some linear equivalents the difference in phase can be significant, although in this specific example it is not.

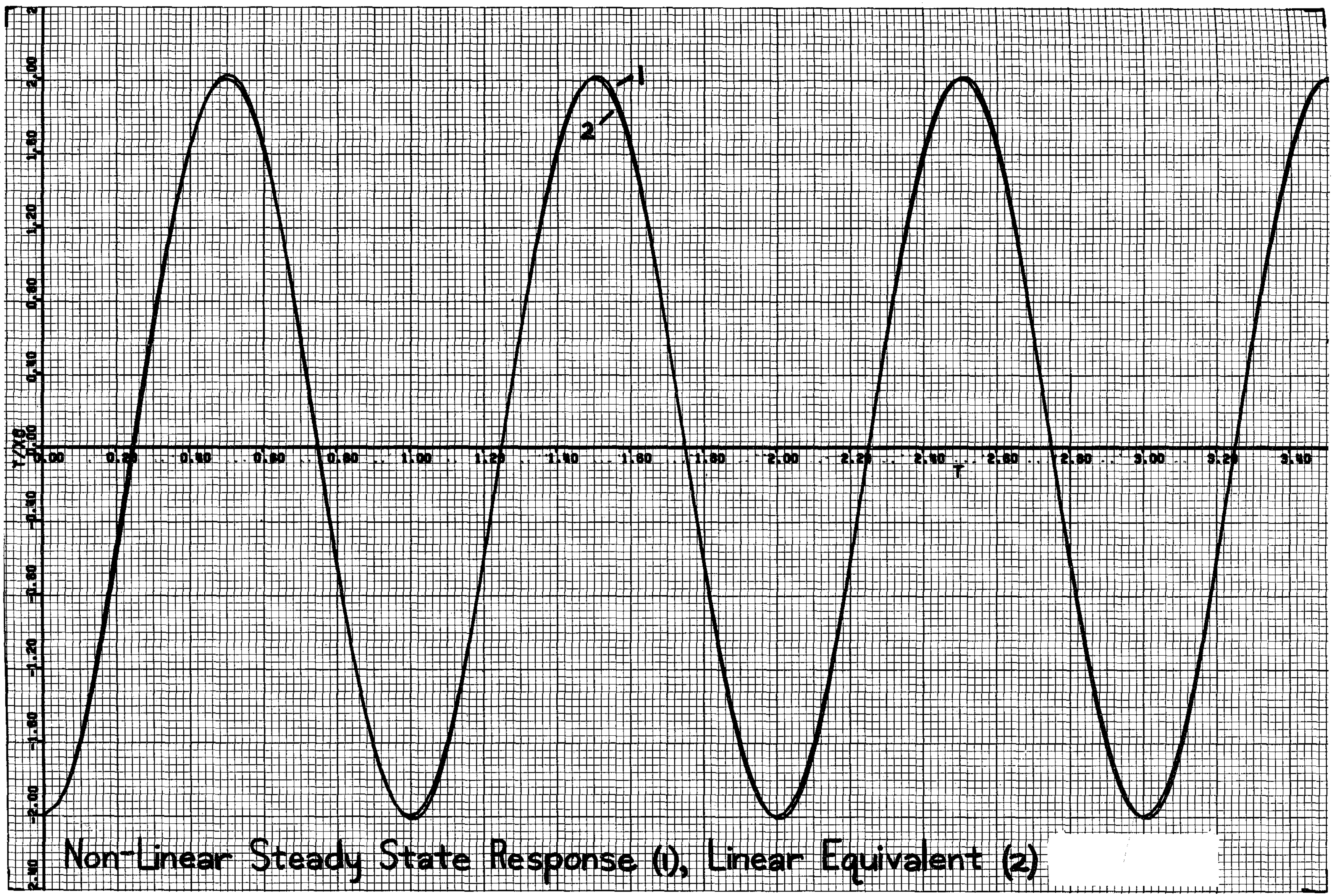


Figure 10. Non-Linear Steady-State Response, Linear Equivalent

VI. EXAMPLES OF APPLICATION

The launch and recovery system examined here could be put to use with the new submersible Sea Cliff. The modified support catamaran Lulu would be the surface vessel. Of the following examples, the first considers the validity of the assumption that the Sea Cliff and cradle could be described as a single degree of freedom system, that responds to the motion of the support vessel but does not influence it. The second and third examples consider two hypothetical situations that might arise in a real operation. The fourth example is an account of the first real application of this analysis to the Alvin salvage operation.

EXAMPLE ONE: A TEST OF THE SINGLE DEGREE OF FREEDOM ASSUMPTION FOR SEA CLIFF AND LULU

Throughout this investigation it has been assumed that a cradle suspended on a spring element could be analyzed as a forced single degree of freedom system. This example considers the validity of this assumption when the submersible is Sea Cliff and the support ship is the Lulu.

PHYSICAL PARAMETERS (Approximate):

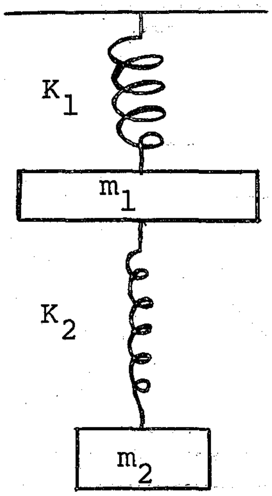
Lulu, a catamaran
Each hull is 96 feet long, 14 feet wide, with an elliptical cross section
Displacement = 450 tons
Weight of added mass = 450 tons
Tons per inch immersion = 6.0 tons per inch
= 72 tons per foot

Sea Cliff
Length = 25 feet
Beam = 10 feet
Displacement = 25 tons
Weight of added mass = 17 tons (for both Sea Cliff & cradle)
Weight of cradle = 10 tons

Connecting Cables
Spring constant, K = 1.0 ton per foot

The added mass estimates can be made by comparing the Lulu and Sea Cliff to similar shapes of the same displacement and of known added mass. The Sea Cliff is compared to an ellipsoid having a length to beam ratio of 2.5 to 1 and a displacement of 25 tons. The Lulu is compared to two rods of elliptical cross section [Ref. 6,9].

The ship and submersible can be modeled with two degrees of freedom. The following diagram depicts this model:



where:
m_1 = 28.0 ton-sec^2/ft
m_2 = 1.617 ton-sec^2/ft
K_1 = 72.0 tons/ft
K_2 = 1.0 ton/ft

Model For Two Degrees of Freedom

FIGURE 11

The two natural frequencies of this system and their corresponding mode shapes can be quite easily determined by hand computation or computer program using well-established methods for solving eigenvalue problems. S.H. Crandall's book, "Engineering Analysis" (Ref. 3), treats this subject quite carefully.

The eigenvalues and eigenvectors for this system are:

Eigenvalues or natural frequencies
ω_1 = 1.6181 ω_2 = 0.7793 radians/second

Eigenvectors or mode shapes

$$X_1 = \begin{matrix} 1.0 \\ -0.309 \end{matrix} \qquad X_2 = \begin{matrix} 0.0179 \\ 1.0 \end{matrix}$$

If we calculate the natural frequency of Lulu without the submersible and if we also compute the natural frequency of the submersible assuming no interaction with the Lulu, then we get the following results:

$$\omega_{Lulu} = \sqrt{K_1/m_1} = 1.604 \text{ rad/sec}$$

$$\omega_{Sub} = \sqrt{K_2/m_2} = 0.786 \text{ rad/sec}$$

The ω_{Lulu} corresponds almost exactly with ω_1, and the ω_{Sub} is nearly identical to ω_2. This supports the assumption that the system could be modeled as a single degree of freedom system.

In addition, the eigenvector, corresponding to ω_1, describes free undamped and undriven motions at the natural frequency of Lulu. The eigenvector shows that the ship and cradle are 180° out of phase, and that the maximum amplitude of the cradle is 0.309 that of the Lulu.

If the cradle is considered a one degree of freedom system, being driven at ω_1, then we can compute the frequency ratio $\omega_1/\omega_2 = 2.075$ and determine the transmissibility from Figure 5. The transmissibility is 0.303 and is very close to that indicated by the eigenvector. For this example, then, the assumption of single degree of freedom motion is certainly valid.

EXAMPLE TWO: SLACK CABLE SITUATION

Under the proper circumstances, it is possible that the relative motions of the surface vessel and submersible bring them close enough together that the cable is completely relaxed. Such a situation is modeled below.

In the original formulation of the differential equation, the origin of coordinates on the cradle was taken to be the equilibrium, or static stretched condition of the cable. Before the equation was non-dimensionalized, it appeared as follows:

$$M \frac{d^2y}{dt^2} = -b\left|\frac{dy}{dt}\right|\frac{dy}{dt} - K(y - X_o \sin(\omega t)).$$

The weight of the cradle in water did not enter into the equation.

However, in the equilibrium position, the cable is stretched by the weight of the cradle suspended on it. Letting W = weight of the cradle in water, then the length the cable is stretched is given by $\Delta L = W/K$. ΔL is now the length the cable can contract before going slack.

If W is both added to and subtracted from the differential equation we get the following result:

$$M \frac{d^2y}{dt^2} = -b\left|\frac{dy}{dt}\right|\frac{dy}{dt} - K(y - W/K - X_o \sin(\omega t)) - W.$$

As long as $y - X_o \sin(\omega t) \leq W/K$, then the cable will not be relaxed; but if $y - X_o \sin(\omega t) > W/K$, then the cable will be relaxed. In a relaxed condition the cradle will feel no spring force; the drag and weight of the cradle will be the only external forces that it feels, and the differential equation during this time reduces to:

$$M \frac{d^2y}{dt^2} = -b\left|\frac{dy}{dt}\right|\frac{dy}{dt} - W.$$

In non-dimensional form this becomes:

$$\frac{d^2Y}{dT^2} = -B\left|\frac{dY}{dT}\right|\frac{dY}{dT} - \frac{4\pi^2}{KX_o} W .$$

$$B = bX_o/M.$$

The slack cable condition can now be summarized in non-dimensional form.

$$\text{If } Y - \sin\left(2\pi \frac{\omega}{\omega_o} T\right) > \frac{W}{KX_o},$$

$$\text{then } \frac{d^2Y}{dT^2} = -B\left|\frac{dY}{dT}\right|\frac{dY}{dT} - \frac{4\pi^2 W}{KX_o} .$$

$$\text{If } Y - \sin\left(2\pi \frac{\omega}{\omega_o} T\right) \leq \frac{W}{KX_o},$$

$$\text{then } \frac{d^2Y}{dT^2} = -B\left|\frac{dY}{dT}\right|\frac{dY}{dT} - 4\pi^2\left(Y - \sin\left(2\pi \frac{\omega}{\omega_o} T\right)\right).$$

It was possible to model this condition on the computer. The differential equation is written into the computer program in such a way that it tests itself for the relaxed condition, and then chooses the proper form of the differential equation. This is given in more detail in Appendix II.

In the example given in Figure 12 the following conditions were used:

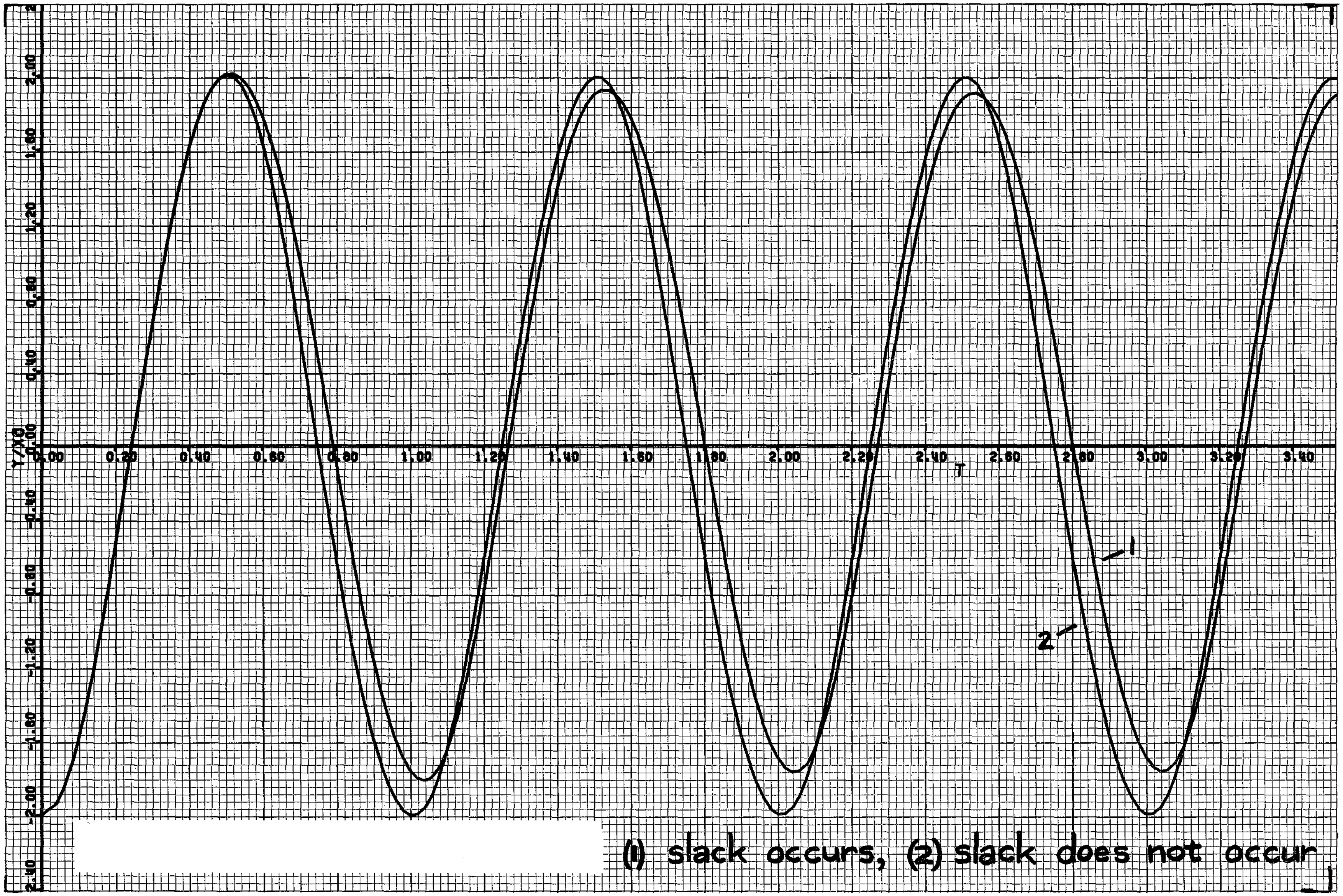
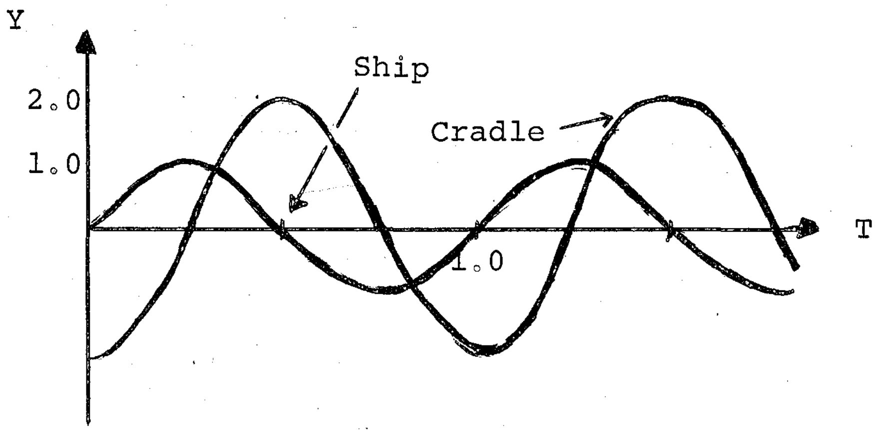


Figure 12. Slack Cable Simulation

$$\omega/\omega_o = 1.0,$$
$$B = 0.3,$$
$$W/KX_o = 1.6.$$

With circumstances that would not permit a slack cable, the transmissibility at steady-state for the prescribed frequency ratio and damping would be $Y_{max} = 2.0$.

This particular example is an easy one to visualize. At $\omega/\omega_o = 1.0$, the motion of the cradle lags that of the ship by 90^o. The following diagram shows these motions plotted together:



Relative Motion of Ship and Cradle

FIGURE 13

The time when the cable might possibly be slack would be when the ship is moving downward and the cradle upwards. In fact, the cradle and ship come nearest to one another when the ship is passing through zero displacement on its way down and the cradle is at its peak upward travel and momentarily is motionless. If this were the case, $X_o \sin(2\pi(\omega/\omega_o)T)$ would be zero, and Y would be 2.0. If W/KX_o were 1.6, the cable would certainly be slack, for a portion of the period. Figure 12 shows the resulting motion. The effect of the slack cable during part of the cycle is to reduce the amount of energy that is supplied to the cradle motion. The presence of damping then reduces the peak to peak excursions of the cradle, but the maximum tensions may increase due to the additional phase lag. The danger of kinking the cable still exists.

EXAMPLE THREE: DOCKING SIMULATION

Docking presents some particular problems. The mass of the cradle is greatly increased when the submersible docks. Two vastly different masses on the cable result in two considerably different natural frequencies. Two different natural frequencies then result in a dynamic response due to two different frequency ratios. An example is described below and presented in Figure 14.

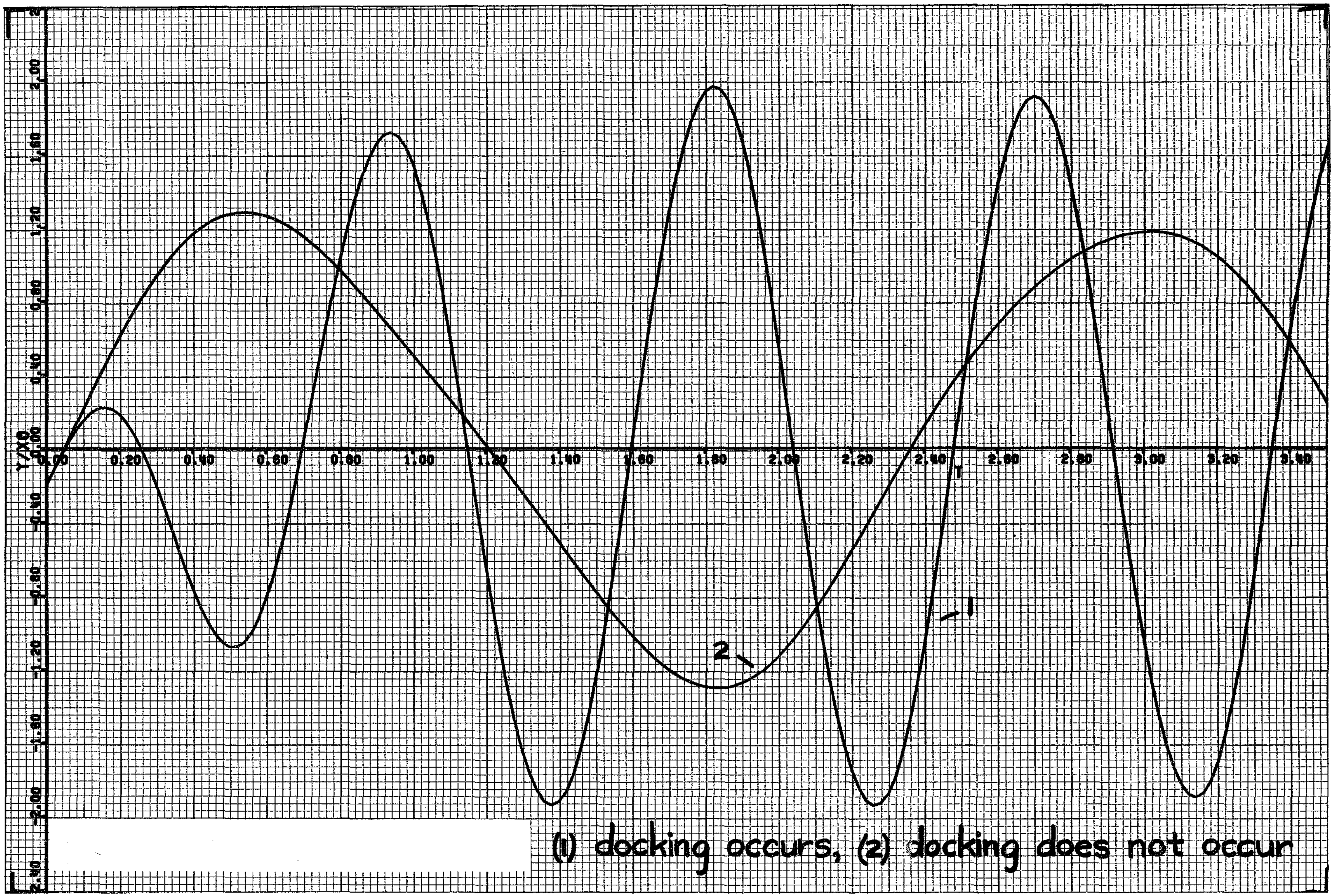


Figure 14. Docking simulation

PHYSICAL PARAMETERS:

Mass + Added Mass:	
Cradle only	= 451 slugs
Cradle + submersible	= 3361 slugs
Spring Constant	= 1000 pounds per foot
Natural Frequencies:	
Cradle only, ω_c	= 1.49 radians per second
Cradle + submersible, ω_s	= 0.545 radians per second

If the surface excitation is due to a wave whose frequency is $\omega = 0.628$, which corresponds to a very common ocean wave with a ten second period, then the frequency ratios for the two cases are:

$$\omega/\omega_c = 0.422,$$

$$\omega/\omega_s = 1.15.$$

Figure 14 shows the response from T = 0.0 of the cradle with and without the submersible on board. This simulates the condition that at time T = 0.0, the submersible contacted the cradle which had been at steady-state. The time axis has been non-dimensionalized with respect to the natural frequency without the cradle on board. Therefore, the record of the cradle and submersible appears to be at a higher frequency. This is not true; the frequencies are the same, but the frequency ratios are different. The scales of amplitude are correct as they appear.

The important feature of this example is that the cradle responds with greater amplitudes after the docking of the submersible.

EXAMPLE FOUR: CALCULATIONS FOR ALVIN SALVAGE OPERATION

This final example is an account of the first practical application of this work. At the time of this writing, August 1, 1969, the Alvin salvage operation is scheduled to begin in approximately ten days. The Alvin is at a depth of 5000 feet and is to be winched to the surface with a synthetic line. The surface ship is to be the Myzar, which is owned and operated by the U.S. Navy, and is of sufficient size to be unaffected by the motion of Alvin.

PHYSICAL PARAMETERS:

Mass Estimate:

Neutrally buoyant displacement of Alvin	=	33,050 lbs.
Lost buoyancy in present state	=	8,100 lbs.
Weight of added mass based on equivalent ellipsoid	=	18,850 lbs.
Total	=	60,000 lbs.
Mass	=	2,200 slugs

Spring Constant: The weight of Alvin in seawater in her flooded condition is approximately 8,100 pounds. This is 15% of the breaking strength (53,000 pounds) of the four-and-one-half-inch circumference Samson nylon line which will be used to recover the Alvin. At this static load, a 100-foot piece of line has a spring constant of K = 1040 pounds per foot.

Earlier it was shown that $K = EA/L$. For this line, $K = 1040(100/L)$ pounds per foot. The natural frequency can now be computed:

$$\omega_o = \sqrt{K/M} = \sqrt{1040(100)/2200L} = 0.689\sqrt{100/L} \text{ rad/sec.}$$

The natural period is then $T = 2\pi/\omega_o$. Natural period versus the length of nylon line is given in Table 1. From this table it is apparent that lengths ranging from 10 to 300 feet have natural periods which correspond to waves commonly found in the sea that could excite the Myzar.

Length of Line (feet)	Period (seconds)
10	2.88
25	4.55
50	6.44
100	9.12
200	12.9
300	15.8
500	20.4
1000	28.8
2000	40.7
5000	64.4

Natural Period Versus Length of Nylon Line
TABLE 1
(For lengths greater than 300 feet, these numbers are useful for qualitative information only.)

If the sea state were such that at some depth a natural frequency were excited, then it is of interest to predict the resulting amplitudes of Alvin's motion. This requires an estimate of drag.

The Alvin is to be hauled up in an upright position. Her cross sectional area, perpendicular to vertical motion is approximately 150 square feet. On page 13 a drag coefficient of $C_D = 1.0$ was cited for Alvin. Using this, and picking a value of $X_o = 4.4$ feet for Myzar's vertical motion, then:

$$b = \tfrac{1}{2}\rho S C_D = 150 \text{ slugs per foot,}$$

$$B = bX_o/M = (150)(4.4)/(2200) = 0.3.$$

From Figure 5, $Y_{max} = 2.14$ at resonance, and therefore $y_{max} = 9.42$ feet for the Alvin. On a short length, the nylon line would break.

In the salvage operation a tensiometer will be used to measure tensions in the line, and an accelerometer will record the motions of the ship. It is hoped that the data will provide a useful comparison to the predictions.

VII. CONCLUSION

A differential equation has been proposed to model the dynamics of a cradle suspended beneath a surface vessel. Several assumptions have been made, and within the limits of these assumptions, the results can be used to predict the motions of objects suspended by elastic members from the surface. The value of this differential equation as a predictive model will not be known until data from model tests or well-instrumented open ocean operations can be obtained.

The Alvin salvage operation may yield some valuable comparative data. In addition, there have been recent model tests conducted at the Naval Ship Research and Development Center in Washington, D.C. These tests were conducted on a model of a submersible and catamaran involved in a recovery operation. The model submersible was connected to the catamaran by spring elements; different surface conditions were generated and the submersible was drawn up to the catamaran at various speeds. The report has not yet been published, but it may provide some useful information.

From a mathematical point of view, a numerical technique has been used to solve a non-linear differential equation. The solution is as good as the technique, which was extremely reliable.

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LIST OF SYMBOLS

F_w	Weight of cradle
F_b	Buoyancy of cradle
F_d	Drag force
F_s	Spring force
M	Virtual mass of cradle and/or submersible
y	Amplitude of cradle
x	Amplitude of ship
X_o	Maximum amplitude of ship
t	Real time
K	Spring constant
E	Modulus of elasticity
A	Cross sectional area of cable
L	Length of cable
C_D	Drag coefficient
S	Cross sectional area of cradle or submersible
ρ	Fluid density
V	Velocity of cradle
b	$1/2\,\rho S C_D$
C	Linear drag coefficient
ω_o	Natural frequency
ω	Driving frequency

B	Non-dimensional damping coefficient, bX_o/M
Y	Non-dimensional response, y/X_o
T	Non-dimensional time, $t/(2\pi/\omega_o)$
ξ	Linear damping ratio, $C/2\omega_o M$
ψ	Phase lag
Y_{max}	Transmissibility, y_{max}/X_o
y_o	Initial condition for y
m_1, m_2	Masses used in example one
K_1, K_2	Spring constants in example one
ω_1, ω_2	Eigenvalues in example one
W	Weight of cradle in water

APPENDIX II
COMPUTER PROGRAM DESCRIPTION AND INSTRUCTIONS FOR USE

SOURCE: The program was adapted for use on the SDS Sigma-7 computer at Woods Hole Oceanographic Institute; the original was obtained from the IBM 1130 Computer Facility in the Mechanical Engineering Department of Massachusetts Institute of Technology. There it was part of a collection of programs known as "Access, Phase I." The following description was taken in part from the description found at the M.I.T. M.E. Computer Facility.

The program was used in two forms, depending on the desired output, printed or plotted. Complete listings of each program follow the description.

SOLUTION OF A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS

1. General
Given the system of first order ordinary differential equations:

$$\frac{dy_1}{dx} = f_1(x, y_1, y_2, \ldots, y_n)$$

$$\frac{dy_2}{dx} = f_2(x, y_1, y_2, \ldots, y_n)$$

$$\vdots$$

$$\frac{dy_n}{dx} = f_n(x, y_1, y_2, \ldots, y_n)$$

and the initial values:

$$y_1(x=0) = y_{1,0} \, ,$$

$$y_2(x=0) = y_{2,0} \, ,$$

$$\vdots$$

$$y_n(x=0) = y_{n,0} \, .$$

The program solves for the y_i using modified fourth order Runge-Kutta formulae. For reference to the mathematical method used, see "RKGS" of the 1130 or 360 IBM Scientific Subroutine Package.

2. Error Control

Control of accuracy and adjustment of the step size, h, is done by generating a test value, δ, based upon a weighted average of the differences of the y_i, as computed with double and single step sizes, 2h and h:

$$\delta = \frac{1}{15} \sum_{i=1}^{n} a_i \; y_i(x_o + 2h) - y_i((x_o + h) + h) \; .$$

If δ is greater than a given tolerance ε_2, the increment h is halved and the procedure to obtain a sufficiently small δ starts again at the point x_o. If δ is less than $\varepsilon_1 = \varepsilon_2/50$, the next step of the integration is carried out with the doubled step size 2h, and subsequently $x_{o_{new}} = x_o + 2h$. However, care is taken in the procedure that the increment never becomes greater than the increment, h_o, specified as an input parameter. The increment of the last step of the procedure is chosen in such a way that the upper limit of the integration is reached exactly.

3. Program Limitations

The maximum number of simultaneous equations that can be solved is 50, for the printed output form, and 10 for the plotted output form. One or more independent groups of equations can be solved at one time, as long as they are all to be evaluated over the same time interval, with the same increment.

4. Sample Problem

On page 19 the single second order non-linear equation was expressed as the following two first order equations:

$$\frac{dY1}{dT} = Y2 \ ,$$

$$\frac{dY2}{dT} = -B|Y2|Y2 - 4\pi^2 (Y1 - \sin(2\pi \frac{\omega}{\omega_0} T)) \ .$$

The problem is to find Y1, Y2, dY1/dT, and dY2/dT for T = 0.0 to 5.0 in increments of 0.02, when B = 0.2 and ω/ω_0 = 0.9.

Function Cards: The two first order equations are punched as follows beginning in column 7:

```
B = 0.2

E = 0.9

DERY(1) = Y(2)

DERY(2) = -B*ABS(Y(2))*Y(2) - 39.48*Y(1)

              + 39.48*SIN(6.283*E*X)
```

E is ω/ω_o and X = T. These cards are placed in Subroutine FCT as shown in the listing, for the program with printed output.

Data Cards: Data may be punched in columns indicated but must have a decimal point.

COLUMN	CONTENTS	VALUE USED IN SAMPLE
Card One		
1-10	Number of sets of data for the same system of equations.	1.0
Card Two		
1-5	Number of equations	2.0
6-10	One number specifying printed output according to these three options. If this number: = 1., the y_i at all points for which the program calculates a value will be printed out (see Error Control). = 2., the points which equal the lower limit of integration + jh_o, where j=1,2..., will be printed out. =3., just the value of the y_i at the upper limit of integration will be printed out. If this option is used, the absolute value of the upper limit of integration must be greater than the lower limit of the interval.	2.0
11-15	Number specifying whether the corresponding derivatives of the y_i will be	1.0

COLUMN	CONTENTS		VALUE USED
	printed out in addition to the values of the y_i: =0., no derivatives will be printed out. =1., derivatives will be printed out at points specified in columns 6-10 of this input card.		
Card Three			
1-10	Lower limit of the interval of integration		0.0
11-20	Upper limit of the interval		5.0
21-30	h_o, initial increment of the independent variable, x. Initial step size and also increment printed if option 2 is followed.		0.02
31-40	$_2$, upper error bound.		0.002
Card Four (Initial Values)			
	Placement of data is within successive blocks of 10 columns, the first block including columns 1-10, etc. If more than 8 blocks are needed, successive cards are used in the same manner, i.e. the 9th block is columns 1-10 of a 2nd card, etc.	Initial values of the y_i. The total number needed is equal to the total number of equations.	-2.4 0.0
Card Five (Error weights)			
	Same as on data cards for initial values	a_i, the error weights. Total must equal 1.	0.5 0.5

Following the first listing is the first page of output for the above example. It is quite self-explanatory.

The second listing is the program that plots the output. The function cards and data cards are prepared in the same way. The program assumes that all dependent variables are labelled consecutively from 1 to 10. It will plot Y(1), Y(3),...Y(9). The particular function cards shown in the listing modeled the slack cable problem, and this program produced the plot in Figure 12.

```
 JØB 64,1100
 LIMIT (TIME,5)
 ASSIGN F:2,(DEVICE,CRA03)
 ASSIGN F:3,(DEVICE,LPA02)
 FØRTRAN GØ
      SUBRØUTINE FCT(X,Y,DERY)
      DIMENSIØN DERY(30),Y(30)
C     FUNCTIØN CARDS FØLLØW THIS CARD
C     NØN-LINEAR DIFFERENTIAL EQUATIØN
C     B IS THE DAMPING CØEFFICIENT
C     E IS THE FREQUENCY RATIØ
      B=0.2
      E=0.9
      DERY(1)=Y(2)
      DERY(2)=-B*ABS(Y(2))*Y(2)-39.48*Y(1)+39.48*SIN(6.283*
     1E*X)
C     FUNCTIØN CARDS PRECEDE THIS CARD
      RETURN
      END
      EXTERNAL FCT,ØUTP
      DIMENSIØN PRMT(5),Y(50),DERY(50),AUX(8,50)
      CØMMØN IPRNT,NDYDX,NSTRT,INIT
      READ(2,1)XNSET
      NSET=XNSET
      WRITE(3,23)NSET
      DØ 9  JJ=1,NSET
      NSTRT=0
      INIT=0
      READ(2,4) XNDIM,XPRNT,XDYDX
      NDIM=XNDIM
      IPRNT=XPRNT
      NDYDX=XDYDX
      READ(2,2)(PRMT(I),I=1,4)
      READ (2,3)(Y(I),I=1,NDIM)
      READ(2,3)(DERY(I),I=1,NDIM)
       IF(JJ-1)21,21,22
 22    WRITE(3,300)
 21   CØNTINUE
      WRITE(3,5)NDIM,(PRMT(I), I=1,4)
      WRITE(3,19)
      DØ 17 I=1,NDIM
      WRITE(3,7)I,Y(I)
 17   CØNTINUE
      WRITE(3,88)
      DØ 18 I=1,NDIM
      WRITE(3,8)I,DERY(I)
 18   CØNTINUE
      WRITE(3,89)
      CALL        RKGSA(PRMT,Y,DERY,NDIM,IHLF,FCT,ØUTP,AUX)
      IF(IHLF-10)9,9,10
 10   IF(IHLF-12)11,12,13
 11   WRITE(3,14)
      GØ TØ 9
```

```
12    WRITE(3,15)
      GO TO 9
13    WRITE(3,16)
9     CONTINUE
      CALL EXIT
1     FORMAT(5E10.0)
2     FORMAT(5E10.2)
3     FORMAT(8E10.2)
4     FORMAT(5E5.0)
5     FORMAT(/5X,'INPUT DATA'//5X,'NUMBER OF EQUATIONS=',1
     11X,I3/5X,'LOWER LIMIT OF X =',6X,E10.2/5X,
     2'UPPER LIMIT OF X =',6X,E10.2/5X,
     3'INITIAL INCREMENT OF X =',E10.2/5X,'ERROR BOUND ='
     4,11X,E10.2/)
7     FORMAT(10X'Y(',I2,')= ',E10.2)
8     FORMAT(10X   'A(',I2,')= ',E10.2)
14    FORMAT(' INITIAL INCREMENT HAS BEEN BISECTED MORE THAN
     1   10 TIMES TO GET SATISFACTORY ACCURACY , THIS SET
     2 OF DATA TERMINATED')
15    FORMAT(1X'INPUT ERROR.  INITIAL INCREMENT = 0.  THIS
     1 SET OF DATA TERMINATED')
16    FORMAT(1X'INPUT ERROR.  INITIAL INCREMENT HAS WRONG
     1 SIGN. THIS SET OF DATA TERMINATED')
19    FORMAT(/5X'INITIAL VALUES OF DEPENDENT VARIABLES')
20    FORMAT(72H1
     1                                                          )
23    FORMAT(/5X'1 INTEGRATION OF SIMULTANEOUS DIFFERENTIAL
     1  EQUATIONS',//6X,'THIS PROGRAM WILL PROCESS ',I3,'
     2 SETS OF INPUT DATA'/)
88    FORMAT(/5X'WEIGHTING COEFFICIENTS FOR ERROR ANALYSIS
     1')
89    FORMAT(/5X'OUTPUT-'/5X'X= VALUE OF INDEPENDENT VARIABLE
     1     '/5X'HALVES= NUMBER OF TIMES INITIAL STEP SIZE
     2 HAS BEEN HALVED'/)
300    FORMAT(1H1)
      END
      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      DIMENSION DERY(50),Y(50)  ,PRMT(5)
      COMMON IPRNT,NDYDX,NSTRT ,INIT
      IF(IPRNT-2)1,2,3
3     IF(X-PRMT(2))10,1,1
2     IF(NSTRT)33,33,34
33    NSTRT=1
      XNEXT=PRMT(1) + PRMT(3)
      XN=1.
      GO TO 1
34    IF(X-XNEXT)10,101,101
101   XN=XN + 1.
      XNEXT=PRMT(1) + XN*PRMT(3)
1     WRITE(3,23)X,IHLF
23     FORMAT(/5X'X= ',E10.3,2X'HALVES= ',I2)
      IF(NDYDX)5,4,5
4     WRITE(3,20)
```

```
20    FØRMAT(4X'I',7X'Y(I)')
      GØ TØ 6
5     WRITE (3,21)
21    FØRMAT(4X'I',9X'Y(I)',5X'DY(I)/DX')
6     DØ 10 I=1,NDIM
      IF(NDYDX)8,9,8
9     WRITE(3,24)I,Y(I)
24    FØRMAT(I5,2XE11.4)
      GØ TØ 10
8     IF(INIT)9,9,41
41    WRITE(3,22)I,Y(I),DERY(I)
22    FØRMAT(I5,2(2XE11.4))
10    CØNTINUE
      INIT=1
      RETURN
      END
      SUBRØUTINE RKGSA(PRMT,Y,DERY,NDIM,IHLF,FCT,ØUTP,AUX)
      DIMENSIØN Y(1),DERY(1),  AUX(8,1),A(4),B(4),C(4),PRM
     1T(5)
      DØ 1 I=1,NDIM
1     AUX(8,I)=.06666667*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)= 0.
      CALL FCT(X,Y,DERY)
      IF(H*(XEND-X))38,37,2
2     A(1)=.5
      A(2)=.2928932
      A(3)=1.707107
      A(4)=.1666667
      B(1)=2.
      B(2)=1.
      B(3)=1.
      B(4)=2.
      C(1)=.5
      C(2)=.2928932
       C(3)=1.707107
      C(4)=.5
      DØ 3 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.
3     AUX(6,I)=0.
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0
4     IF((X+H-XEND)*H)7,6,5
5     H=XEND-X
6     IEND=1
7     CALL ØUTP(X,Y,DERY,IREC,NDIM,PRMT)
```

```
      IF(PRMT(5))     40,8,40
8     ITEST=0
9     ISTEP=ISTEP+1
      J=1
10    AJ=A(J)
      BJ=B(J)
      CJ=C(J)
      DO 11 I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
11    AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF(J-4)12,15,15
12    J=J+1
      IF(J-3)13,14,13
13    X=X+.5*H
14    CALL FCT(X,Y,DERY)
      GO TO 10
15    IF(ITEST)16,16,20
16    DO 17 I=1,NDIM
17    AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
18    IHLF=IHLF+1
      X=X-H
      H=.5*H
      DO 19 I=1,NDIM
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
19    AUX(6,I)=AUX(3,I)
      GO TO 9
20    IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
21     CALL FCT(X,Y,DERY)
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
22    AUX(7,I)=DERY(I)
      GO TO 9
23    DELT=0.
      DO 24 I=1,NDIM
24    DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF(DELT-PRMT(4))28,28,25
25    IF(IHLF-10)26,36,36
26    DO 27 I=1,NDIM
27    AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GO TO 18
28    CALL FCT(X,Y,DERY)
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
```

```
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
 29   DERY(I)=AUX(7,I)
      CALL ØUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)
      IF(PRMT(5)) 40,30,40
 30   DØ 31 I=1,NDIM
      Y(I)=AUX(1,I)
 31   DERY(I)=AUX(2,I)
      IREC=IHLF
      IF(IEND)32,32,39
 32   IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      IF(IHLF) 4,33,33
 33   IMØD=ISTEP/2
      IF(ISTEP-IMØD-IMØD)4,34,4
 34   IF(DELT-.02*PRMT(4))35,35,4
 35   IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      GØ TØ 4
 36   IHLF=11
      CALL FCT(X,Y,DERY)
      GØ TØ 39
 37   IHLF=12
      GØ TØ 39
 38   IHLF=13
 39   CALL ØUTP(X,Y,DERY,IHLF,NDIM,PRMT)
 40   RETURN
      END
 LØAD (LMN,X),(GØ),(UNSAT,(F4LIB))
 RUN (LMN,X)
 DATA
1.0
2.0  2.0  1.0
0.0       5.0       .02       .002
-2.4      0.0
0.5       0.5
```

```
INTEGRATION OF SIMULTANEOUS DIFFERENTIAL EQUATIONS

THIS PROGRAM WILL PROCESS    1 SETS OF INPUT DATA


INPUT DATA

NUMBER OF EQUATIONS                      2
LOWER LIMIT OF X =                .00E 00
UPPER LIMIT OF X =                .25E 01
INITIAL INCREMENT OF X =          .20E-01
ERROR BOUND =                     .20E-02


INITIAL VALUES OF DEPENDENT VARIABLES
     Y( 1)=    -.24E 01
     Y( 2)=     .00E 00

WEIGHTING COEFFICIENTS FOR ERROR ANALYSIS
     A( 1)=     .50E 00
     A( 2)=     .50E 00

OUTPUT
X= VALUE OF INDEPENDENT VARIABLE
HALVES= NUMBER OF TIMES INITIAL STEP SIZE HAS BEEN HALVED


X=    .000E 00  HALVES=  0
I        Y(I)        DY(I)/DX
1  -.2400E 01
2   .0000E 00

X=    .400E-01  HALVES=  0
I        Y(I)        DY(I)/DX
1  -.2323E 01    .3887E 01
2   .3887E 01    .9753E 02

X=    .600E-01  HALVES=  0
I        Y(I)        DY(I)/DX
1  -.2226E 01    .5810E 01
2   .5810E 01    .9425E 02

X=    .800E-01  HALVES=  0
I        Y(I)        DY(I)/DX
1  -.2091E 01    .7638E 01
2   .7638E 01    .8813E 02

X=    .100E 00  HALVES=  0
I        Y(I)        DY(I)/DX
1  -.1921E 01    .9319E 01
2   .9319E 01    .7962E 02
```

```
JOB 64,1100
LIMIT (TIME,5)
MESSAGE USES PLOT TAPE
ASSIGN F:2,(DEVICE,CRA03)
ASSIGN F:3,(DEVICE,LPA02)
ASSIGN F:PLOT,(DEVICE,7T),(BIN)
FORTRANH LS,GO
      SUBROUTINE FCT(X,Y,DERY)
      DIMENSION DERY(50),Y(50)
C     FUNCTION CARDS FOLLOW THIS CARD
C     SLACK CABLE SIMULATION
C     B IS THE DAMPING COEFFICIENT
C     E IS THE FREQUENCY RATIO
C     W IS THE NON-DIMENSIONALIZED EQUILIBRIUM STRETCH IN
C     THE CABLE
      W=1.6
      A=0.0
      B=0.3
      C=1.0
      D=1.0
      E=1.0
      F=SIN(6.283*E*X)
      IF(Y(1)-F-W)2,2,1
1     A=-39.48*W
      D=0.0
      C=0.0
    2 CONTINUE
      DERY(1)=Y(2)
      DERY(2)=-B*ABS(Y(2))*Y(2)-39.48*C*Y(1)+39.48*D*F +A
      DERY(3)=Y(4)
      DERY(4)=-B*ABS(Y(4))*Y(4)-39.48*Y(3)+39.48*F
C     FUNCTION CARDS PRECEDE THIS CARD
      RETURN
      END
      EXTERNAL FCT,OUTP
      DIMENSION PRMT(5),Y(50),DERY(50),AUX(8,50),S(252,10)
     1,T(252)
      COMMON IPRNT,NDYDX,NSTRT,INIT
      READ(2,1)XNSET
      NSET=XNSET
      WRITE(3,23)NSET
      DO 9  JJ=1,NSET
      NSTRT=0
      INIT=0
      READ(2,4) XNDIM,XPRNT,XDYDX
      NDIM=XNDIM
      IPRNT=XPRNT
      NDYDX=XDYDX
      READ(2,2)(PRMT(I),I=1,4)
      READ (2,3)(Y(I),I=1,NDIM)
      READ(2,3)(DERY(I),I=1,NDIM)
       IF(JJ-1)21,21,22
22     WRITE(3,300)
```

```
21    CONTINUE
      WRITE(3,5)NDIM,(PRMT(I), I=1,4)
      WRITE(3,19)
      DO 17 I=1,NDIM
      WRITE(3,7)I,Y(I)
17    CONTINUE
      WRITE(3,88)
      DO 18 I=1,NDIM
      WRITE(3,8)I,DERY(I)
18    CONTINUE
      WRITE(3,89)
      CALL DESET
      CALL       RKGSA(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      IF(IHLF-10)9,9,10
10    IF(IHLF-12)11,12,13
11    WRITE(3,14)
      GO TO 9
12    WRITE(3,15)
      GO TO 9
13    WRITE(3,16)
9     CONTINUE
      CALL EXIT
1     FORMAT(5E10.0)
2     FORMAT(5E10.2)
3     FORMAT(8E10.2)
4     FORMAT(5E5.0)
5     FORMAT(/5X,'INPUT DATA'//5X,'NUMBER OF EQUATIONS=',1
     11X,I3/5X,'LOWER LIMIT OF X =',6X,E10.2/5X,
     2'UPPER LIMIT OF X =',6X,E10.2/5X,
     3'INITIAL INCREMENT OF X =',E10.2/5X,'ERROR BOUND ='
     4,11X,E10.2/)
7     FORMAT(10X'Y(',I2,')= ',E10.2)
8     FORMAT(10X   'A(',I2,')= ',E10.2)
14    FORMAT(' INITIAL INCREMENT HAS BEEN BISECTED MORE THAN
     1   10 TIMES TO GET SATISFACTORY ACCURACY . THIS SET
     2 OF DATA TERMINATED')
15    FORMAT(1X'INPUT ERROR.  INITIAL INCREMENT = 0.  THIS
     1 SET OF DATA TERMINATED')
16    FORMAT(1X'INPUT ERROR.  INITIAL INCREMENT HAS WRONG
     1 SIGN. THIS SET OF DATA TERMINATED')
19    FORMAT(/5X'INITIAL VALUES OF DEPENDENT VARIABLES')
20    FORMAT(72H1
     1                                    )
23    FORMAT(/5X'1 INTEGRATION OF SIMULTANEOUS DIFFERENTIAL
     1  EQUATIONS',//6X,'THIS PROGRAM WILL PROCESS ',I3,'
     2 SETS OF INPUT DATA'/)
88    FORMAT(/5X'WEIGHTING COEFFICIENTS FOR ERROR ANALYSIS
     1')
89    FORMAT(/5X'OUTPUT-'/5X'X= VALUE OF INDEPENDENT VARIABLE
     1     '/5X'HALVES= NUMBER OF TIMES INITIAL STEP SIZE
     2 HAS BEEN HALVED'/)
300    FORMAT(1H1)
      END
```

```
C
      SUBROUTINE RKGSA(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      DIMENSION PRMT(5),Y(50),DERY(50),AUX(8,50),S(252,10)
     1,T(252)
      DIMENSION A(4),B(4),C(4)
      DO 1 I=1,NDIM
 1    AUX(8,I)=.06666667*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)= 0.
      CALL FCT(X,Y,DERY)
      IF(H*(XEND-X))38,37,2
 2    A(1)=.5
      A(2)=.2928932
      A(3)=1.707107
      A(4)=.1666667
      B(1)=2.
      B(2)=1.
      B(3)=1.
      B(4)=2.
      C(1)=.5
      C(2)=.2928932
       C(3)=1.707107
      C(4)=.5
      DO 3 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.
 3    AUX(6,I)=0.
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0
 4    IF((X+H-XEND)*H)7,6,5
 5    H=XEND-X
 6    IEND=1
    7 CALL OUTP (X,Y,DERY,IREC,NDIM,PRMT,S,T)
      IF(PRMT(5))     40,8,40
 8    ITEST=0
 9    ISTEP=ISTEP+1
      J=1
 10   AJ=A(J)
      BJ=B(J)
      CJ=C(J)
      DO 11 I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
 11   AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF(J-4)12,15,15
```

```
12    J=J+1
      IF(J-3)13,14,13
13    X=X+.5*H
14    CALL FCT(X,Y,DERY)
      GO TO 10
15    IF(ITEST)16,16,20
16    DO 17 I=1,NDIM
17    AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
18    IHLF=IHLF+1
      X=X-H
      H=.5*H
      DO 19 I=1,NDIM
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
19    AUX(6,I)=AUX(3,I)
      GO TO 9
20    IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
21     CALL FCT(X,Y,DERY)
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
22    AUX(7,I)=DERY(I)
      GO TO 9
23    DELT=0.
      DO 24 I=1,NDIM
24    DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF(DELT-PRMT(4))28,28,25
25    IF(IHLF-10)26,36,36
26    DO 27 I=1,NDIM
27    AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GO TO 18
28    CALL FCT(X,Y,DERY)
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
29    DERY(I)=AUX(7,I)
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT,S,T)
      IF(PRMT(5)) 40,30,40
30    DO 31 I=1,NDIM
      Y(I)=AUX(1,I)
31    DERY(I)=AUX(2,I)
      IREC=IHLF
      IF(IEND)32,32,39
32    IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
```

```
      IF(IHLF) 4,33,33
 33   IMØD=ISTEP/2
      IF(ISTEP-IMØD-IMØD)4,34,4
 34   IF(DELT-.02*PRMT(4))35,35,4
 35   IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      GØ TØ 4
 36   IHLF=11
      CALL FCT(X,Y,DERY)
      GØ TØ 39
 37   IHLF=12
      GØ TØ 39
 38   IHLF=13
   39 CALL ØUTP(X,Y,DERY,IHLF,NDIM,PRMT,S,T)
      DIMENSIØN IBUF(1000)
      CALL PLØTS(IBUF,-1000)
      CALL PLØT(0.0,15.0,-3)
      CALL AXIS(0.0,0.0,'T',-1,25.0,0.0,0.0,0.2)
      CALL AXIS(0.0,-10.0,'Y/XØ',4,20.0,90.0,-4.0,0.4)
      CALL PLØT(0.0,-10.0,-3)
      DØ 41 I=1,NDIM,2
      S(251,I)=-4.0
      S(252,I)=0.4
      T(251)=0.0
      T(252)=0.2
      CALL LINE(T(1),S(1,I),250,1,0,0)
   41 CØNTINUE
      CALL PLØT(25.0,0.0,999)
      CALL EXIT
 40   RETURN
      END
C
      SUBRØUTINE ØUTP (X,Y,DERY,IHLF,NDIM,PRMT,S,T)
      DIMENSIØN PRMT(5),Y(50),DERY(50),AUX(8,50),S(252,10)
     1,T(252)
      CØMMØN IPRNT,NDYDX,NSTRT,INIT
      IF(IPRNT-2)1,2,3
 3    IF(X-PRMT(2))10,1,1
 2    IF(NSTRT)33,33,34
 33   NSTRT=1
      XNEXT=PRMT(1) + PRMT(3)
      XN=1.
      GØ TØ 1
 34   IF(X-XNEXT)10,101,101
 101  XN=XN + 1.
      XNEXT=PRMT(1) + XN*PRMT(3)
    1 CØNTINUE
    6 J=J+1
      DØ 11 I=1,NDIM
   11 S(J,I)=Y(I)
      T(J)=X
 10   CØNTINUE
```

```
       INIT=1
       RETURN
       ENTRY DESET
       J=0
       RETURN
       END
 LOPE (LMN,X),(GO),(UNSAT,(3))
 RUN (LMN,X)
 DATA
1.0
4.0  2.0  1.0
0.0       5.0       .02       .002
-2.0      0.0       -2.0      0.0
0.5       0.5
 EOD
```

November 1969

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Woods Hole Oceanographic Institution
Reference NO. 69-88

DYNAMIC ANALYSIS OF A LAUNCH AND RECOVERY SYSTEM FOR A DEEP SUBMERSIBLE by John Kim Vandiver. 76 pages. December 1969. Contract No. Nonr 3484(00); NR 260-107.

An analysis of the motion of a docking platform for a deep submersible is presented. The problem is defined as a docking platform, or cradle, suspended beneath a surface vessel by elastic elements, composed of cable, chain, or similar material. The analysis attempts to predict the motion of the cradle in response to sinusoidal motion of the surface vessel.

The physical system is mathematically modeled as a nonlinear differential equation of the form:

$$M \frac{d^2y}{dt^2} + b \frac{dy}{dt}\frac{dy}{dt} + Ky = KXo \sin(\omega t)$$

This assumes a single degree of freedom system, which models the cradle as a mass connected by a linear spring to a vessel moving vertically in a sinusoidal fashion. The damping is non-linear and is proportional to velocity squared.

This problem is solved for transient as well as steady-state conditions by numerical techniques. An equivalent linear differential equation is proposed. The results are applied to four examples. Three of them are directly related to a possible application of the launch and recovery system to the new deep submersible SEA CLIFF and her support ship LULU. The fourth example gives an account of the application of the results to the salvage operation of the deep submersible ALVIN.

1. Alvin Salvage Operation
2. Computer Program
3. Docking Cradle
4. LULU
5. MIT
6. Office of Naval Research
7. SEA CLIFF Launch and Recovery
8. Sinusoidal Surface Vessel Motion

I. Daubin, S.C.
II. Vandiver, J.K.
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2. Computer Program						
3. Docking Cradle						
4. LULU						
5. MIT						
6. Office of Naval Research						
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